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World gold prices and stock returns in China: insights for hedging and diversification strategies

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Abstract
In this paper we make use of several multivariate GARCH models to investigate both return and volatility spillovers between world gold prices and stock market in China over the period from March 22, 2004 through March 31, 2011. We also analyze the optimal weights and hedge ratios for gold-stock portfolio holdings and show how empirical results can be used to build effective diversification and hedging strategy. Our results show evidence of significant return and volatility cross effects between gold prices and stock prices in China. In particular, past gold returns play a crucial role in explaining the dynamics of conditional return and volatility of Chinese stock market and should thus be accounted for when forecasting future stock returns. Our portfolio analysis suggests that adding gold to a portfolio of Chinese stocks improves its risk-adjusted return and helps to effectively hedge against stock risk exposure over time. Finally, we show that the VAR-GARCH model performs better than the other multivariate GARCH models.

Keywords: Stock markets, gold prices, diversification and hedging effectiveness, GARCH models

JEL classification: G12, F3, Q43

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1. Introduction

The purpose of the present study is threefold. Our first objective is to examine the dynamic return and volatility transmission using a bivariate VAR–GARCH model for gold and stock markets in China. The empirical model is particularly advantageous in that it allows simultaneous shock transmission in the conditional returns and volatilities. We then use the model’s results to compute and analyze the optimal weights and hedge ratios for gold-stock portfolio holdings. We finally assess the diversification and hedging effectiveness of gold in China based on different competing multivariate GARCH-based models.

The literature on gold and other precious metal markets has recently regained particular attention from finance researchers and practitioners. Such a tendency can straightforwardly be explained by the investors’ willingness to produce a hedge to diversify away the increasing risk in the stock markets through investing in other asset classes. Indeed, the high volatility and widespread contagion caused by successive financial turbulences and crises over the last decades have prompted investors to consider alternative investment instruments as a part of diversified portfolios of stocks. Not only oil asset but also major precious metals including gold emerged as natural desirable asset classes eligible for portfolio diversification because they offer different volatilities and returns of lower correlations with stocks, at both sector and market levels (Arouri and Nguyen, 2010; Daskalaki and Skiadopoulos, 2011). It is now common that when risk aversion mounts due, for example, to increasing instability and uncertainty in stock markets or to long swings in the price of oil, most investors are directed towards the metals markets and gold in particular, being viewed as the refugee or safe haven asset in time of crises.

It is thus not surprising that the existing literature on the gold asset is part of the one examining the price dynamics, stochastic properties and roles of the commodities markets in portfolio management. A number of studies have questioned the responses of precious metal prices to changes in international institutional and macroeconomic factors (e.g., Kaufman and Winters, 1989; Christie–David et al., 2000; Heemskerk, 2001; Ciner, 2001; Batten et al., 2010). Sjaastad and Scacciavillani (1996) find, for example, that fluctuations of floating exchange rates of major currencies, following the breakdown of the Bretton Woods currency arrangements, have led to price instability in the world gold market over the period from January 1982 to December 1990. For their part, Batten et al. (2010) document the sensitivity of precious metals volatility to macroeconomic factors, but with different degrees. The overall results suggest that precious metals are too distinct to be considered a single asset class. In
particular, gold volatility can be empirically explained by monetary variables. Gold also seems to be highly sensitive to exchange rate and inflation, which implies that it can offer the best hedge during inflationary pressures and exchange rate fluctuations and an optimal portfolio of precious metals that minimizes risk should be dominated by gold (Hammoudeh et al., 2011).

More recent studies have rather looked at the issues of gold price volatility modeling and information transmission between precious metals and other commodities in order to draw the implications of the estimated results for portfolio diversification and hedging strategies involving precious metals.¹ For example, Hammoudeh and Yuan (2008) make use of various GARCH-based models to examine the properties of conditional volatility for three important metals (gold, silver, and copper) while controlling for shocks from world oil prices (WTI) and three-month US Treasury bill interest rate. Using daily three-month futures prices of the three metals, they find that conditional volatility of gold and silver is more persistent, but less sensitive to leverage effects than that of copper. This finding leads to suggest, on the one hand, the importance of accurate volatility modeling especially when gold and silver are used as underlying assets in financial derivatives contracts, and on the other hand the valuable contribution of these two metals in down markets and crisis times. Hammoudeh et al. (2010) document, from a multivariate VARMA-GARCH model, weak volatility spillovers across precious metals, but strong sensitivity of metal volatility to exchange rate variability. They further point out the role of gold as a hedge against exchange rate risk when optimal weights and hedge ratios are computed.

While understanding the dynamic interactions between gold price changes and stock markets is a crucial element for portfolio designs, risk management and asset pricing, these interactions have only been examined very recently by, to the best of our knowledge, two studies. Baur and Lucey (2010) show that gold serves as a safe haven for stocks in the US, the UK, and Germany especially after extreme negative shocks affecting stock markets. Gold is also a hedge for stocks in the US and the UK. Baur and McDermott (2010) test whether gold represents a safe haven against stocks of major emerging and developing countries, and show that over the period 1979 to 2009 gold is both a hedge and a safe haven for major European stock markets and the US but not for Australia, Canada, Japan and large emerging markets such as the BRIC countries. Furthermore, gold is found as a strong safe haven for most developed markets during severe episodes of the recent financial crisis.

¹ See Arouri et al. (2012) for a detailed discussion of this literature.
Our study extends the existing literature into return and volatility spillover between gold and stock markets, portfolio designs in the presence of both gold and stocks, and choice of appropriate models for modeling gold-stock interactions. Specifically, we provide a thorough analysis of how shocks and volatility are transmitted from world gold market to Chinese stock market and from Chinese stock market to world gold market. This research is of particular interest for several reasons. First, none of the previous studies has considered the volatility spillover between gold and stock markets while a better understanding of the transmission mechanisms between them helps building accurate stock valuation models and accurate forecasts of the volatility of both markets. Empirical results from volatility spillover analysis also permit to address several important issues such as hedging strategies, optimal portfolio allocation, and derivatives management with respect to the uncertainties associated with gold price fluctuations. Note that the works of Baur and Lucey (2010) and Baur and McDermott (2010) only provide grounds for understanding the gold-stock return relationships. Second, the linkages between world gold prices and Chinese stock market have never been examined despite the increasing role of China in the world economy. According to the IMF statistics, China showed, over the last decade, a very high economic growth rate ranging from 8.3% (2001) and 14.2% (2007). This strong economic performance made China the second largest economy in 2010 with respect to the total GDP measured at purchasing power parity. In the meantime, the study of Lai and Tseng (2010) suggests that the Chinese stock market is not only a safe haven but also a hedge for global investors’ portfolios, in particular during times of financial turbulence. Additionally, we note the continuing surge in China’s gold demand in recent years. The World Gold Council’s report on May 17, 2012 pointed out that China’s investment demand for physical gold is more than doubled from 40.7 tons a year ago to 90.9 tons in the first quarter of 2012. Soaring international market prices, high uncertainties in stock and property markets, and rising inflation expectations are undoubtedly candidate factors that have led to the robust growth of the domestic gold investment market in China. Like anywhere in the world, gold may thus be seen as a good alternative investment and the best hedge to inflation. Finally, our empirical analysis relies on the multivariate specification of the vector autoregressive - generalized autoregressive conditional heteroskedasticity model (VAR-GARCH) developed by Ling and McAleer (2003). This model offers the possibility to explore the conditional volatility dynamics of the return series as well as the conditional cross effects and volatility spillover between them. It also provides meaningful estimates of the model’s parameters with fewer computational complications than other multivariate GARCH specifications, such as the full-factor GARCH model. Furthermore, the findings can be used
to analyze the diversification and hedging effectiveness across gold asset and stock market. Some papers have taken the VAR-GARCH approach to investigate the volatility spillover and hedging strategies between Gulf Arab equity sectors (Hammoudeh et al., 2009), between previous metals and exchange rates (Hammoudeh et al., 2010), between crude oil spot and futures returns of the Brent and WTI oil price benchmarks (Chang et al., 2011), and between oil and stock markets (Arouri et al., 2011). These studies commonly suggest the suitability of the VAR-GARCH model for capturing the dynamic linkages between different asset markets as well as building optimal portfolios.

Overall, we find evidence of significant volatility cross effects between world gold price and stock market in China over the period 2004-2011. In particular, past gold shocks are found to play a crucial role in explaining the time-varying patterns of conditional volatility of Chinese stock returns and should thus be accounted for when making volatility forecasts of future stock returns. On the other hand, our portfolio analysis suggests that adding the gold asset to a well-diversified portfolio of Chinese stocks improves its risk-adjusted performance and that stock risk exposures can be effectively hedged using gold. Moreover, we show that the VAR-GARCH model performs better than four alternative multivariate volatility models (CCC-, DCC-, scalar and diagonal BEKK-GARCH) since it provides higher risk-adjusted return performance and hedging effectiveness.

The remainder of this article is structured as follows. Section 2 introduces our empirical methodology. Section 3 presents the sample data and their stochastic properties. Section 4 discusses the empirical findings and shows their implications on optimal portfolio designs and risk management. Section 5 provides some concluding remarks.

2. Empirical method

Since their apparition, the ARCH/GARCH-family models have received a particular attention from researchers and practitioners as far as the issue of volatility modeling and forecasting of macroeconomic and financial variables is addressed. When the objective is to investigate volatility interdependence and spillovers between different time-series, multivariate GARCH specifications such as the CCC-GARCH model of Bollerslev (1990), the BEKK-GARCH model of Engle and Kroner (1995) or the DCC-GARCH model of Engle (2002) should be more relevant than univariate models. Empirical results reported in Hassan and Malik (2007), Agnolucci (2009), and Kang et al. (2009), among others, confirm effectively the superiority
of these models and show that they satisfactorily capture the stylized facts of commodities’ conditional volatility as well as the dynamics of volatility interactions.

Given that we attempt to examine the shock and volatility spillover between gold prices and stock market in China as well as to derive the implications of the results on optimal weights and hedge ratios for gold-stock portfolio holdings, the abovementioned multivariate GARCH models are naturally suitable for our research question. However, these models are involved with an excessive number of parameters when the number of variables in the system is important. They can also encounter convergence problems during the estimation process, especially when additional exogenous variables are introduced into the conditional mean and variance equations. These reasons lead us to employ the VAR-GARCH model developed by Ling and McAleer (2003). This model typically combines a multivariate GARCH process and a VAR model. Its main advantage rests on its flexibility to model the conditional mean cross-effects and conditional volatility transmission between the series under consideration with fewer computational complexities than other multivariate volatility models. The ability of the VAR-GARCH model to capture cross-market volatility interactions has been confirmed by recent research (Chang et al., 2011; Arouri et al., 2011).

In this study, we use the bivariate VAR-GARCH as baseline model to shed light on the diversification and hedging effectiveness of a portfolio involving the gold asset and stocks in China. We also consider four competing bivariate volatility models (CCC-GARCH, DCC-GARCH, diagonal BEKK-GARCH and scalar BEKK-GARCH) for comparison purpose.

2.1 Bivariate VAR(1)-GARCH(1,1)

We use the VAR(1)-GARCH(1,1) model to accommodate the return and volatility interdependencies between gold and stock markets. The conditional mean of our empirical model is specified as follows

\[
\begin{aligned}
    R_t &= \mu + \Phi R_{t-1} + \epsilon_t, \\
    \epsilon_t &= H_t^{1/2} \eta_t,
\end{aligned}
\]

where \( \mu \) is the vector of constant terms in the VAR, \( R_t = (r_t^g, r_t^s)' \) is the vector of returns on the stock market index and the gold price index, respectively. \( \Phi \) refers to a \((2 \times 2)\) matrix of

\footnote{Using the univariate and bivariate AIC and BIC information criteria to choose the optimal lag length of the univariate GARCH(\(p,q\)) process and then that of the bivariate VAR-GARCH model for the pair of gold and stock markets, our results select one lag for both conditional mean and variance equations. They thus lead us to choose the bivariate VAR(1)-GARCH(1,1) specification.}
coefficients. \( \varepsilon_i = (\varepsilon_i^s, \varepsilon_i^g)' \) is the vector of the error terms of the conditional mean equations for stock and gold returns respectively. \( \eta_i = (\eta_i^s, \eta_i^g)' \) refers to a sequence of independently and identically distributed (i.i.d) random errors; and \( H_t = \begin{pmatrix} h_t^s & h_t^{sg} \\ h_t^{gs} & h_t^g \end{pmatrix} \) is the conditional variance-covariance matrix of stock and gold returns. \( h_t^s, h_t^g \) and \( h_t^{sg} \) are specified as follows:

\[
\begin{align*}
    h_t^s &= C_s^2 + \beta_{s1}^2 \times h_{t-1}^s + \alpha_{s1}^2 \times (\varepsilon_{t-1}^s)^2 + \beta_{s2}^2 \times h_{t-1}^g + \alpha_{s1}^2 \times (\varepsilon_{t-1}^g)^2 \\
    h_t^g &= C_g^2 + \beta_{g1}^2 \times h_{t-1}^g + \alpha_{g1}^2 \times (\varepsilon_{t-1}^g)^2 + \beta_{g2}^2 \times h_{t-1}^s + \alpha_{g1}^2 \times (\varepsilon_{t-1}^s)^2 
\end{align*}
\] (2) (3)

It is obvious from Eq. (2) and Eq. (3) that the volatility transmission across the gold and stock markets over time comes from two sources: (i) the cross values of error terms, \((\varepsilon_t^s)^2\) and \((\varepsilon_t^g)^2\), which capture the impact of direct effects of shock transmission, and (ii) the cross values of lagged conditional volatilities, \(h_t^{sg}\) and \(h_t^{gs}\), which directly account for the risk transfer between markets. The stationary condition requires that the roots of the equation \( |I_2 - AL - BL| = 0 \) must be outside the unit circle, where \( L \) is a lag polynomial, \( I_2 \) is a \((2 \times 2)\) identity matrix, and

\[
A = \begin{pmatrix} \alpha_{s1}^2 & \alpha_{s2}^2 \\ \alpha_{g1}^2 & \alpha_{g2}^2 \end{pmatrix} \text{ and } B = \begin{pmatrix} \beta_{s1}^2 & \beta_{s2}^2 \\ \beta_{g1}^2 & \beta_{g2}^2 \end{pmatrix}
\]

Let \( \rho \) be the constant conditional correlation, the conditional covariance between stock and gold returns is modelled as:

\[
h_t^{sg} = \rho \times \sqrt{h_t^s} \times \sqrt{h_t^g}
\] (4)

According to the above specifications, our empirical model simultaneously allows long-run volatility persistence as well as shock and volatility transmissions between gold and stock markets under consideration. Even though the assumption of constant conditional correlation may be viewed as restrictive given changing economic conditions, the extension to dynamic conditional correlation is not possible for instance because the statistical properties of such a model have not been analyzed at the theoretical level yet. We estimate the parameters of the above bivariate model by quasi-maximum likelihood estimation (QMLE), which is robust to any departure from normality conditions. See Ling and McAleer (2003) for further details on necessary and sufficient conditions regarding the consistency of QMLE for the model used.
2.2 Competing bivariate GARCH(1,1) models

We define again $R_t = (r_t^s, r_t^g)'$ as the vector of the returns on stock market index and gold price index, and $H_t = \begin{pmatrix} h_t^s & h_t^{s,g} \\ h_t^{g,s} & h_t^g \end{pmatrix}$ as the conditional variance-covariance matrix of stock and gold returns. We specify the conditional mean of the bivariate AR(1)-GARCH(1,1) as follows

$$
\begin{align*}
R_t &= \mu + \Phi R_{t-1} + \varepsilon_t \\
\varepsilon_t &= H_t^{1/2} \eta_t 
\end{align*}
$$

(5)

where $H_t^{1/2}$ is a $(2 \times 2)$ symmetric positive definite matrix and $\eta_t = (\eta_t^s, \eta_t^g)'$ is the vector of i.i.d. random errors with $E(\eta_t) = 0$ and $Var(\eta_t) = I_2$.

We further assume that $H_t$ follows a bivariate GARCH(1,1) process. Different specifications for $H_t$ thus lead to different multivariate GARCH-type models. As we stated earlier, the most popular multivariate parameterizations include the full BEKK-GARCH model of Engle and Kroner (1995), the CCC-GARCH of Bollerslev (1990) and the DCC-GARCH of Engle (2002).

The full BEKK-GARCH which imposes positive definiteness restrictions for $H_t$ is given by

$$
H_t = C' C + A' \varepsilon_{t-1} \varepsilon_{t-1}' A + B' H_{t-1} B
$$

(6)

where $C$ is a $(n \times n)$ upper triangular matrix, $A$ and $B$ are $(n \times n)$ coefficient matrices. $C' C$ is the decomposition of the intercept matrix. Each element $(i, j)th$ in $H_t$ depends on the corresponding $(i, j)th$ element in $(\varepsilon_{t-1}, \varepsilon_{t-1})$ and $H_{t-1}$. Accordingly, past shocks and volatility are allowed to directly spill over from a market to another, and they are captured by coefficients of $A$ and $B$ matrices. Since the BEKK-GARCH model suffers from the curve of dimensionality and may encounter convergence problem, researchers often use its two restricted versions: the diagonal BEKK-GARCH and the scalar BEKK-GARCH.

The diagonal parameterization of the BEKK-GARCH(1,1) is obtained by imposing the diagonality of the parameter matrices $A$ and $B$. The advantage of this diagonalization is that the model is guaranteed to be positive definite and requires the estimation of fewer parameters than the full-rank parameterization. Consequently, the conditional variance and covariance processes take the following forms:
\[
\begin{align*}
  h_t^s &= C_s + \alpha_s (\varepsilon_{t-1}^s)^2 + \beta_s h_{t-1}^s \\
  h_t^g &= C_g + \alpha_g (\varepsilon_{t-1}^g)^2 + \beta_g h_{t-1}^g \\
  h_{tg} &= C_{tg} + \alpha_{tg} \varepsilon_{t-1}^s \varepsilon_{t-1}^g + \beta_{tg} h_{t-1}^g 
\end{align*}
\]

(7)

where \( h_t^s \) and \( h_t^g \) are the conditional variances of \( r_t^s \) and \( r_t^g \) respectively. Eq. (7) indicates that direct volatility transmission between gold and stock returns is not possible because the conditional volatility of each market depends only on its own past shocks and past volatility.

The diagonal BEKK-GARCH model is covariance stationary under the following conditions:

\[ \alpha_s^2 + \beta_s^2 < 1, \ \alpha_g^2 + \beta_g^2 < 1, \ \text{and} \ |\alpha_g + \beta_s| < 1. \]

The scalar BEKK-GARCH is more restricted than the diagonal BEKK-GARCH. It is derived from the full-rank BEKK-GARCH by imposing that the parameter matrices \( A \) and \( B \) are both equal to the product between a scalar and an identity matrix. Its formal representation is given by

\[ H_t = C' C + a^2 \varepsilon_{t-1} \varepsilon_{t-1} + b^2 H_{t-1} \]

(8)

The CCC-GARCH model of Bollerslev (1990) and the DCC-GARCH model of Engle (2002) belong to another class of multivariate GARCH processes where the time-varying conditional covariances are parameterized to be proportional to the product of the corresponding conditional standard deviations. Here, the objective is to obtain an intuitive and meaningful economic interpretation of the conditional correlation coefficients.

Applying to our research question, the variance-covariance matrix of the bivariate CCC-GARCH(1,1) model is defined as follows:

\[ H_t = D_t PD_t \]

(9)

where \( D_t = \text{diag}(\sqrt{h_t^s}, \sqrt{h_t^g}) \), and \( P = (\rho_{ij}) \) refers to a \((2\times2)\) matrix that contains the constant conditional correlations \( \rho_{ij} \) with \( \rho_i = 1, \forall i = s, g \). The conditional variances and covariance are given by

\[
\begin{align*}
  h_t^s &= C_s + \alpha_s (\varepsilon_{t-1}^s)^2 + \beta_s h_{t-1}^s \\
  h_t^g &= C_g + \alpha_g (\varepsilon_{t-1}^g)^2 + \beta_g h_{t-1}^g \\
  h_{tg} &= \rho \sqrt{h_t^s h_t^g} 
\end{align*}
\]

(10)
Bollerslev (1990) shows that the positiveness of the coefficients associated with ARCH and GARCH terms is indeed not required to obtain a positive definite matrix $P$. The model is covariance stationary when the roots of $\text{det}(I_2 - \lambda A - \lambda B) = 0$ are outside the unit circle of the complex plan, where $I_2$ is a (2×2) identity matrix and

$$A = \begin{pmatrix} \alpha_s & 0 \\ 0 & \alpha_g \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} \beta_s & 0 \\ 0 & \beta_g \end{pmatrix}$$

Engle (2002) proposes the DCC-GARCH(1,1) model that allows the conditional correlation matrix to vary over time as follows

$$P_t = (\text{diag}(Q_t))^{-1/2} Q_t (\text{diag}(Q_t))^{-1/2}$$

where $Q_t = (q''_{it})$ is a (2×2) symmetric positive definite matrix given by

$$Q_t = (1 - \alpha - \beta)\overline{Q} + \alpha \eta_{t-1} \eta'_{t-1} + \beta Q_{t-1}$$

In Eq. (12), $\alpha$ and $\beta$ are non-negative scalars such that $\alpha + \beta < 1$, and $\overline{Q}$ is a (2×2) matrix of unconditional correlations of the standardized errors $\eta_t$. The conditional variances are specified as univariate GARCH(1,1) processes, similar to those of the CCC-GARCH model. Engle (2002) indicates that the specification of the dynamic conditional correlation structure presents no obstacle to model estimation.

Overall, the bivariate GARCH-type models presented above do not explicitly allow for shock and volatility cross-effects between gold and stock market returns, as compared to the VAR-GARCH model. It is thus expected that our benchmark model provides a better description of the dynamic linkages between markets of interest and allows for more accurate decisions regarding portfolio diversification and hedging effectiveness.

3. Data and preliminary analysis

Our sample data consist of daily time series of the Chinese stock market and 3-month gold futures prices (GOLD3M) over the period from March 22, 2004 to March 31, 2011. The MSCI China Index which is a free-float weighted equity index developed with a base value of 100 as of December 31 1992 by Morgan Stanley Capital International is used to represent the overall performance of the Chinese stock market. The gold futures, widely traded at COMEX (Commodity Exchange) in New York, are used to represent the world gold market.
fluctuations. Their prices are measured in US dollars per troy ounce. Data were extracted from MSCI and Bloomberg databases. We compute the return series by taking the differences in the logarithm of two consecutive prices.

Figure 1. Stock and gold price dynamics

The time-paths followed by the MSCI China index and gold futures prices are depicted in Figure 1. We see that the Chinese stock market shares common phases of market dynamics with the world stock market. It experienced continuing attractiveness since the end of 2004 until the end of 2007, then showed significant decreases due to the effects of the global financial crisis between the beginning of 2008 and the mid-2009, and started to recover afterwards. The evolution of gold futures prices is particularly marked by an increasing trend over the whole study period and their sharp increases over the recent years. The common wisdom of the financial community is that the role of gold as an alternative investment as well as a hedge instrument during financial markets’ turbulent times has contributed to drive up their prices.

Table 1 presents selected descriptive statistics for the return series. The gold market provided higher daily returns than the Chinese stock market over our study period. The severe impacts of the recent global financial crisis 2007-2009 seem to be the main reason that explains the underperformance of the Chinese stock market as well as the performance of the world gold market. The unconditional volatility, as measured by standard deviations, is substantially higher for the stock market than for the gold market. As a result, gold was a better investment with higher risk-adjusted returns. This finding thus suggests that gold might be a good hedge for portfolios of stocks, especially in times of crises and bear market periods.

We also estimated our empirical models with gold spot, also traded at COMEX in New York, but the results remain intact. These results can be made entirely available under request addressed to corresponding author. Note finally that the correlation between gold spot and gold futures is 0.853 over the study period.
Table 1. Descriptive statistics and stochastic properties of return series

<table>
<thead>
<tr>
<th></th>
<th>MSCI China</th>
<th>Gold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (%)</td>
<td>0.057</td>
<td>0.067</td>
</tr>
<tr>
<td>Maximum (%)</td>
<td>14.044</td>
<td>8.625</td>
</tr>
<tr>
<td>Minimum (%)</td>
<td>-12.836</td>
<td>-7.574</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>2.010</td>
<td>1.261</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.04</td>
<td>-0.131</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>9.459</td>
<td>7.202</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>3187.202***</td>
<td>1354.089***</td>
</tr>
<tr>
<td>Q(15)</td>
<td>37.346***</td>
<td>43.728***</td>
</tr>
<tr>
<td>Q’(15)</td>
<td>1870.689***</td>
<td>378.195***</td>
</tr>
<tr>
<td>ARCH(15)</td>
<td>38.061***</td>
<td>40.761***</td>
</tr>
<tr>
<td>ADF</td>
<td>-42.002***</td>
<td>-41.760***</td>
</tr>
<tr>
<td>PP</td>
<td>-42.002***</td>
<td>-41.785***</td>
</tr>
</tbody>
</table>

Notes: The table reports the basic statistics of return series as well as their stochastic properties. Jarque-Bera is the empirical statistics of the Jarque-Bera test for normality based on skewness and excess kurtosis. Q(15) and Q’(15) are the empirical statistics of the Ljung-Box tests for autocorrelations applied to return and squared return series. ARCH refers to the empirical statistics of the Engle (1982) test for conditional heteroscedasticity. ADF and PP are the empirical statistics of the Augmented Dickey-Fuller and Phillips-Perron unit root tests. We present the model with a constant and a trend, but the results are similar when models without a constant and/or a trend are used. ’*’ and ’***’ indicate the rejection of the null hypotheses of normality, no autocorrelation, no ARCH effects, and unit root at the 5% and 1% levels, respectively.

With negative skewness coefficients and kurtosis coefficients above three, the distributions of Chinese stock market and gold returns are typically asymmetric, and the probability of observing large negative returns is higher than that of a normal distribution. The Jarque-Bera test statistics clearly confirm the rejection of the null hypothesis of normality for both return series. The results from the Ljung-Box test indicate evidence of autocorrelation in return and squared return series for both gold and stock markets. The ARCH test for conditional heteroscedasticity provides strong evidence of ARCH effects in both return series, which consequently suggests the usefulness and suitability of GARCH-type models for examining volatility dynamics and transmission of gold and stock markets in China. Finally, two commonly-used unit root tests, Augmented Dickey-Fuller (ADF) test and Phillips-Perron (PP) test, are performed in order to examine the stationarity property of the series considered. The results indicate that return series are stationary, and thus they can be straightforwardly used for further analysis.

4. Results and portfolio implications

This section discusses the estimation results of our bivariate VAR(1)-GARCH(1,1) model for gold returns and stock market returns in China. We particularly focus on the extent of volatility transmission between the markets considered. The competing models (CCC-GARCH, DCC-GARCH, diagonal BEKK-GARCH, and scalar BEKK-GARCH) are also
estimated, but the results are not shown here in order to conserve space. Finally, we compare the diversification and hedging effectiveness across models.

4.1 Volatility cross-effects in gold and stock markets in China

Table 2 presents the estimation results of our bivariate VAR(1)-GARCH(1,1) model, together with statistical tests applied to standardized residuals. Taking a close look at the estimates of the mean equations, we find that own autoregressive parameters are not significant for both markets under consideration. The one-period lagged gold returns are found to significantly affect the current returns of Chinese stock market at the 1% level. This implies that one can predict Chinese stock returns by using past gold returns over time. Some recent papers also document that Chinese stock market is not weak-form efficient almost all the time (see, e.g., Groenwold et al., 2004; Gao and Kling, 2005). Inversely, past stock market returns in China has no impact on gold returns.

Table 2. Estimates of VAR(1)-GARCH(1,1) model for gold returns and stock market returns in China

<table>
<thead>
<tr>
<th></th>
<th>MSCI China</th>
<th>Gold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Const(μ)</td>
<td>0.084</td>
<td>0.067***</td>
</tr>
<tr>
<td></td>
<td>(0.032)</td>
<td>(0.024)</td>
</tr>
<tr>
<td>Stock(-1)</td>
<td>0.023</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>Gold(-1)</td>
<td>0.240***</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>Const(ν)</td>
<td>0.138***</td>
<td>0.091***</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>(ε^2)</td>
<td>0.295***</td>
<td>0.050**</td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>(0.023)</td>
</tr>
<tr>
<td>(ε^2)</td>
<td>0.105</td>
<td>0.184***</td>
</tr>
<tr>
<td></td>
<td>(0.055)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>h^2</td>
<td>0.947***</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(7.238)</td>
</tr>
<tr>
<td>h^2</td>
<td>0.107</td>
<td>0.977***</td>
</tr>
<tr>
<td></td>
<td>(0.088)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>β</td>
<td></td>
<td>0.137***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.022)</td>
</tr>
</tbody>
</table>

LogL -6257.688   AIC 6.872   SIC 6.923

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>JB</td>
<td>59.333***</td>
<td>668.278</td>
</tr>
<tr>
<td>Q(15)</td>
<td>30.572++</td>
<td>12.733</td>
</tr>
<tr>
<td>Q²(15)</td>
<td>19.937</td>
<td>42.675++</td>
</tr>
<tr>
<td>ARCH(15)</td>
<td>14.665</td>
<td>13.168</td>
</tr>
</tbody>
</table>

Notes: Stock(-1), Gold(-1), and β are the one-period lagged returns of Chinese stock market and gold market, and conditional constant correlation, respectively. Const(μ) and Const(ν) are the constant terms in the conditional mean and variance equations. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively. The AIC and BIC criteria measure the relative goodness of fit of the estimated model. JB, Q(15), Q²(15), and ARCH(15) refer to the empirical statistics of the Jarque-Bera test for normality, the Ljung-Box tests for autocorrelations of order 15 applied to standardized residuals and squared standardized residuals, and the Engle (1982) test for conditional heteroscedasticity of order 15, respectively. *, **, and *** indicate the rejection of the null hypothesis of associated statistical tests at the 10%, 5%, and 1% levels, respectively.
As to the coefficients associated with ARCH and GARCH terms, which capture shock and volatility independence in the conditional variance equations, we find common patterns for both gold returns and stock returns in China. First, these coefficients appear to be highly significant. Second, the estimated conditional volatility series do not change very rapidly under the effects of return innovations given the small size of ARCH coefficients. They tend instead to develop gradually over time with respect to substantial effects of past volatility, advocated by the large values of GARCH coefficients. Accordingly, investors and fund managers seeking profit from trading gold and Chinese stocks may consider active investment strategies based on volatility persistence and current market trends. It would be advisable, for example, to increase the amount of portfolio investment if markets are actually rising and to decrease it if they are falling, all while keeping in mind that the viability of such strategies depends on the stability and the strength of performance between successive periods.

The extent of volatility transmission between gold market and Chinese stock market is particularly interesting. Our findings show that past gold shocks play a crucial role in explaining the time-dynamics of conditional volatility of stock returns in China and should thus be accounted for when making volatility forecasts of future stock returns. Moreover, the conditional volatility of gold returns is significantly affected by unpredicted changes in the returns on the MSCI China index, \( \epsilon_{t-1}^g \). A shock to Chinese stock market, regardless of its sign, thus implies an increase in the volatility of gold returns. On the other hand, past volatility of Chinese stock market, represented by \( h_{t-1}^s \), has no significant effects on the gold return volatility. As for the opposite direction, the impact of past gold volatility, \( h_{t-1}^g \), on the conditional volatility of Chinese stock market is also statistically insignificant.

Turning out to the estimates of the constant conditional correlation (CCC), we find that Chinese stock market returns are positively correlated with gold returns, but the cross-market correlation is relatively weak (0.137). One may expect, in view of this finding, substantial gains from having both assets in the same portfolio. Moreover, it is obvious that the significant volatility cross-effects we show previously require portfolio managers and investors to quantify the optimal weights and hedging ratios to properly deal with the comovement between gold and Chinese stock returns.

Lastly, the results of diagnostic tests based on standardized residuals show that the departure from normality and the serial correlation are greatly reduced and that there are no
ARCH effects, as compared to test statistics we report for the returns in Table 1. Thus, the bivariate VAR(1)-GARCH (1,1) model offers a flexible way to capture the joint dynamics of gold returns and Chinese stock market returns.

4.2 Optimal portfolio designs and hedging ratios in the presence of gold

We now show the implications of our previous results on gold-stock optimal portfolio. To this end, we consider a hedged portfolio made up of gold and Chinese stock market index in which an investor seeks to protect himself from exposure to stock price movements by investing in the gold asset. In practice, the investor’s objective is to minimize the risk of his gold-stock portfolio while keeping the same expected returns. Following Kroner and Ng (1998), we determine the optimal holding weight of gold in a one-dollar portfolio of stock/gold at time \( t \), denoted by \( w_t'' \), as follows:

\[
\frac{h_t' - h_t''}{h_t' - 2h_t'' + h_t''} = w_t''
\]

where \( h_t' \), \( h_t'' \) and \( h_t''' \) refer respectively to the conditional volatility of the gold returns, the conditional volatility of the Chinese stock market index returns and the conditional covariance between gold and stock returns at time \( t \). These series are estimated from our benchmark bivariate VAR(1)-GARCH(1,1) model and four competing bivariate GARCH-based models we presented in Section 2. When short selling is not allowed, the mean-variance portfolio optimization approach imposes the following constraints on the optimal weight of gold:

\[
w_t'' = \begin{cases} 
0, & \text{if} \quad w_t'' < 0 \\
w_t'', & \text{if} \quad 0 \leq w_t'' \leq 1 \\
1, & \text{if} \quad w_t'' > 1 
\end{cases}
\]

From Eq. (13), the proportion of wealth that the investor put on the Chinese stock market index is \( 1 - w_t'' \).

Now if the objective of our investor is to optimally hedge the risk of his investment in stock markets, he should take an appropriate position on the gold market so that it minimizes the risk of the hedged portfolio. Concretely, a long position (buying) of one dollar on the stock segment must be hedged by a short position (selling) of \( \beta_{t,ss} \) dollars on the gold asset. Following Kroner and Sultan (1993), the optimal hedge ratio \( \beta_{t,ss} \) can be expressed as
\[ \beta^*_t = \frac{h^*_t}{h^*_t} \]

The average values of realized optimal weights \( w^*_t \) and optimal hedge ratios \( \beta^*_t \) are reported in Table 3. A glance at the coefficients shows that the optimal weight for the gold asset in the hedged portfolios is 0.330 according to our VAR(1)-GARCH(1,1) model and varies from 0.322 to 0.333 with respect to competing bivariate GARCH-based models. The highest optimal weight for gold of 0.333 is obtained for the CCC-GARCH and DCC-GARCH models, while the scalar BEKK-GARCH model provides the lowest one (0.322). Overall, our results show that, to minimize the risk without lowering the expected return of the gold-stock portfolio, the investor operating in Chinese stock markets should hold more stocks than gold. This finding can be effectively explained by the low correlation of gold returns with stock returns in China, and then simply adding a small portion of gold to a diversified portfolio of Chinese stocks will reduce substantially its overall risk for a given level of expected return.

**Table 3. Optimal weights and hedge ratios for portfolio of gold and stocks in China**

<table>
<thead>
<tr>
<th>Model</th>
<th>VAR-GARCH</th>
<th>CCC-GARCH</th>
<th>DCC-GARCH</th>
<th>Scalar BEKK-GARCH</th>
<th>Diagonal BEKK-GARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w^*_t )</td>
<td>0.330</td>
<td>0.333</td>
<td>0.333</td>
<td>0.322</td>
<td>0.325</td>
</tr>
<tr>
<td>( \beta^*_t )</td>
<td>0.197</td>
<td>0.192</td>
<td>0.194</td>
<td>0.178</td>
<td>0.188</td>
</tr>
</tbody>
</table>

Notes: The table reports average optimal weight of gold and hedge ratios for an gold-stock portfolio using conditional variances and covariance estimated from our benchmark model and four competitive volatility-spillover models: VAR(1)-GARCH(1,1), diagonal BEKK-GARCH(1,1), DCC-GARCH(1,1), CCC-GARCH(1,1), and scalar BEKK-GARCH(1,1). For all models considered, the conditional mean equations contain a constant term and an autoregressive component.

The average optimal hedge ratios, \( \beta^*_t \), are generally low and do not exceed 0.2. This finding has several important implications for short hedgers. First, the low hedge ratios suggest that stock investment risk can simply be hedged by taking short positions in the gold futures markets. The largest ratio is obtained from the VAR-GARCH model (0.197), followed closely by the DCC-GARCH and the CCC-GARCH. This implies that one dollar long (buy) in the Chinese stock market index should require the investor to go short 19.7 cents in the gold futures market. The lowest hedge ratio is obtained in the case of the scalar BEKK-GARCH model (0.178), meaning that the investor must sell the smallest quantity of gold in the gold futures market. Altogether, our findings for optimal hedge ratios support with the view that the gold asset should be an integral part of a diversified portfolio of stocks and help reduce the risk of the hedged portfolio, particularly in periods of crisis.
4.3 Diversification and hedging effectiveness

In subsection 4.2 we show, based on the estimations of selected bivariate GARCH models, that adding the gold asset in the diversified portfolio of stocks leads to improve its risk-adjusted performance. One may, however, wonder how effective are the diversification and the hedging associated with the gold asset? For this reason, we propose to run portfolio simulations to empirically examine the diversification and hedging effectiveness of gold. These portfolio simulations are based on our optimal portfolio weights and hedging ratios. More precisely, we use the estimates of our VAR-GARCH and four competing bivariate GARCH models to build two distinct portfolios: a portfolio composed of Chinese stocks only (PF I) and a gold-stock portfolio with the optimal weights calculated in subsection 4.2 (PF II). For each model, the effectiveness of the portfolio diversification is evaluated by comparing the realized risk and return characteristics of the considered portfolios (PF I vs. PF II). In the meantime, the effectiveness of hedging across constructed portfolios can be assessed by examining the realized hedging errors ($HE$) which are determined as follows (Ku et al., 2007)

$$HE = \left( \frac{Var_{\text{unhedged}} - Var_{\text{hedged}}}{Var_{\text{unhedged}}} \right)$$  \hspace{1cm} (15)

where $Var_{\text{hedged}}$ refers to the variance of the hedged portfolio’s returns (PF II), and $Var_{\text{unhedged}}$ the variance of the unhedged portfolio’s returns (PF I). Eq. (15) indicates that the higher the $HE$ ratio, the more effective the hedging in terms of portfolios’ variance reduction. The model that generates the highest $HE$ ratio is the best one to be used for building gold-stock hedging strategy.

| Table 4. Portfolio designs and diversification in presence of the gold asset |
|------------------|------------------|------------------|------------------|
|                  | Mean             | Standard         | Realized risk-adjusted returns ($\times 100$) |
|                  |                  | deviation        |                                |
| PF I             | 0.0562           | 2.0113           | 2.7962                        |
| PF II – VAR-GARCH| **0.0596**       | 1.4358           | **4.1510**                    |
| PF II – Diagonal BEKK-GARCH | 0.0595        | 1.4531           | 4.0975                        |
| PF II – Scalar BEKK-GARCH | 0.0595       | 1.4576           | 4.0830                        |
| PF II – DCC-GARCH | **0.0596**      | 1.4415           | 4.1364                        |
| PF II – CCC-GARCH | **0.0596**      | 1.4415           | 4.1364                        |

Notes: This table compares the realized risk-adjusted returns, measured by relating each portfolio’s mean to its standard deviation, of different portfolios. Figures in boldface indicate the highest mean, standard deviation, and risk-return trade-off. PF I is a portfolio of 100% stocks. PF II is a weighted gold-stock portfolio in which the weights are given by the optimal weights reported in Table 4.

The results from portfolio simulations in Table 4 show that adding the gold asset to the diversified portfolio of stocks improves its risk-adjusted return ratios regardless of the models.
used. More interestingly, our benchmark VAR-GARCH model provides the best risk-adjusted return ratio, closely followed by the DCC and CCC-GARCH models.

Table 5. Hedging effectiveness

<table>
<thead>
<tr>
<th></th>
<th>Variance (%)</th>
<th>Hedge effectiveness (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PF I</td>
<td>4.0454</td>
<td></td>
</tr>
<tr>
<td>PF II – VAR-GARCH</td>
<td><strong>2.0615</strong></td>
<td><strong>49.0409</strong></td>
</tr>
<tr>
<td>PF II – Diagonal BEKK-GARCH</td>
<td>2.1116</td>
<td>47.8018</td>
</tr>
<tr>
<td>PF II – Scalar BEKK-GARCH</td>
<td>2.1245</td>
<td>47.4843</td>
</tr>
<tr>
<td>PF II – DCC-GARCH</td>
<td>2.0778</td>
<td>48.6373</td>
</tr>
<tr>
<td>PF II – CCC-GARCH</td>
<td>2.0778</td>
<td>48.6373</td>
</tr>
</tbody>
</table>

Notes: This table reports the portfolio variance and hedge effectiveness ratios, computed using Equation (14). Numbers in boldface indicate the hedged portfolio with lowest variance and the highest variance reduction. PF I is a portfolio of 100% stocks. PF II is a weighted gold-stock portfolio in which the weights are given by the optimal weights reported in Table 4.

Table 5 presents the hedging effectiveness (HE) ratios for the models we consider. The results show that hedging strategies involving stocks and gold assets lead to reduce considerably the portfolio’s risk (variance), as compared to the risk of the portfolio made up of stocks only. The variance reduction ranges from 47.48% for the scalar BEKK-GARCH model to 49.04% for the VAR-GARCH model. This finding, consistent with the analysis of portfolio designs and diversification, thus consolidates the dominant position of the VAR-GARCH model as it helps reducing the largest part of the initial portfolio’s variance. The diagonal and scalar BEKK-GARCH models are the worst models in terms of portfolio’s variance reduction. Chang et al. (2011) reach similar conclusion when they use the same models to calculate optimal portfolio weights and optimal hedge ratios for the crude oil spot and futures returns of two major crude oil benchmarks, Brent and WTI.

6. Conclusion

In this paper, we investigate the extent of volatility transmission, portfolio designs, and hedging effectiveness in gold and stock markets in China. We employ the recent VAR-GARCH modeling approach, which allows for volatility spillover cross effects, we find significant volatility transmission between Chinese stock market and world gold market. The examination of optimal weights and hedge ratios suggests that optimal portfolios should have stocks outweigh gold assets and that the stock investment risk can be hedged with relatively low hedging costs by taking a short position in the gold futures markets. In particular, we show that optimally hedged gold-stock portfolios outperform traditional portfolios of stocks regardless of bivariate volatility models, and that our baseline VAR-GARCH model is the best performing model in terms of both diversification and hedging effectiveness. On the
whole, gold assets can be considered a dynamic and valuable asset class that helps to improve the risk-adjusted performance of a well-diversified portfolio of stocks.

References


