Trade Liberalization and Optimal R&D Policies with Process Innovation

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Abstract

We set up a theoretical framework to discuss the impact of trade liberalization and R&D policies on domestic exporting firms’ incentive to innovate and social welfare. In this framework, exporting firms invest in R&D to reduce their production costs and, in return, receive R&D subsidies from the government. While firms target at maximizing their profits, the government aims to maximize the social welfare. We consider different settings of firm competition to explore their strategic behaviours as well as the government’s strategic behaviour at the policy stage. We find that tradeliberalization in the foreign market is always welfare enhancing and, in most cases, leads to higher export sales and R&D investments of firms, and raises productivity at firms and industry level. When firms are independent monopolies in the overseas market, it is optimal for the government not to provide any R&D subsidy. When goods are close substitutes, the social optimum can be achieved as a Nash equilibrium by applying an optimal R&D tax. Trade liberalization induces a higher R&D tax rate to be levied on firms. When firms also conduct business in the home market, it is always optimal for the government to provide firms with a financial
support to their R&D activity. While this R&D subsidy is decreasing in the trade cost when firms are independent monopolies, its monotonicty in the trade costs is determined by the convexity of the R&D cost function when firms produce close substitutes.

**Keywords:** Trade, R&D, subsidies, welfare

**JEL classification:** F12, F13, F15, O31

### 1 Introduction

In our globalized world, opening up to trade is the key trend in economic development. International trade enlarges the potential size of the market and brings gains from trade. However, it also creates more challenges to firms as they will face fiercer competition not only from home but also from overseas. To survive and develop in the market place, firms need to improve their productivity and in that process, innovation is essential. From a policy standpoint, a government can support their domestic exporting firms by providing them with either export or R&D subsidies. However, as export subsidies are often restricted due to international agreements, providing subsidies to firms’ R&D activities become the most effective policy tool of any national governments nowadays. Several studies such as Spencer and Brander (1983), Bagwell and Staiger (1994), Brander (1995), Neary and Leahy (2000), and Leahy and Neary (2001) even find that subsidizing R&D is more powerful than subsidizing exports.

Clearly, trade liberalization and R&D policies are closely related. While trade liberalization affects factors impacting innovation activities such as market size and toughness of competition, R&D investment determines the benefits of undertaking the trade. It is surprising that not much has been done to examine the links between these two policy factors although there exists rich branches of literature studying each factor separately. Filling this gap will be the main task of this paper. In doing so, this paper considers the issue of exporting duopoly in a basic model of strategic R&D with trade liberalization occuring in an exporting market. Here, firms produce horizontally differentiated products and invest in R&D to reduce their marginal cost of production. Government policies include providing a subsidy to the exporting firms to stimulate their R&D activity. However, it should be noted that the main aim of the government policies is not only to expand firms’ output sales (in the overseas and/or home market) but also to maximize
domestic welfare. This environment creates a two-stage game which can be solved by backward induction. In the first stage, the government decides on how much to subsidize R&D activity of firms in order to maximize domestic welfare. In the second stage, firms maximize their profits by choosing export volumes/domestic sales as well as level of R&D investment optimally taking into account the subsidy rate provided by the government and the other firm’s action. The result at the end of the second stage is a Cournot-Nash equilibrium. Depending on the setting environment, the strategic behaviours of the government and firms are different and convey different implications for the optimal R&D subsidy. However, overall, common findings are that trade liberalization is always welfare enhancing as it helps firms further expand their output sales, both overseas and at home. In most cases, trade liberalization encourages firms to undertake more cost-reducing R&D by enlarging their profit margins. This, in turn, improve firms’ and industry productivity.

The first results are developed in a simple setting with two domestic exporting firms competing in an overseas market. Foreign firms are assumed either non-existent or are too small to count on. When these firms produce completely independent products, the best policy from welfare maximizing point of view for the government is to provide firms with zero R&D support. This is because each firm is already a monopoly in its own product line. By capturing the whole market segment of its own, the firm enjoys the highest level of profit. Subsidizing firms’ R&D does not increase firms’ profits net of R&D subsidy costs so welfare is unchanged. When goods are close substitutes, the government’s optimal policy turns out to be taxing firms’ R&D activity instead of subsidizing it. This is because too much competition between domestic firms in the foreign market will erode the power that the home country as a whole can exercise in the foreign market. This optimal R&D tax increases when trade liberalization in the foreign market occurs. In this case, trade liberalization has no impact on R&D investments of firms. Another result is that when goods are less differentiated, the R&D tax rate tends to be higher.

Results on optimal R&D subsidy turn out to be significantly different when exporting firms also conduct business at home. The first-best policy is to subsidize R&D of firms even when they are independent monopolies. This is due to consumer-surplus motive of subsidizing R&D as domestic consumers will gain very much from having access to different varieties. Trade liberalization implemented by the foreign market induces a higher optimal R&D subsidy level when goods are completely independent because the
extra gain from undertaking further R&D outweighs its cost. However, when firms produce close substitutes, the monotonicity of this R&D subsidy in the trade cost is not immediately conclusive. In particular, it depends on the convexity of the R&D cost function. If the R&D cost function is not so convex, the marginal benefit from doing R&D is greater than its cost so the optimal R&D subsidy increases. By contrast, if the R&D cost function is very convex, R&D becomes extremely costly so the optimal R&D policy should discourage R&D through cutting down the level of R&D subsidy. In addition, an increase in the degree of substitutability of goods decreases optimal R&D subsidy.

In characterizing R&D subsidies, a majority of existing studies (e.g. Brander, 1995; Neary and Leahy, 2000; Leahy and Neary, 2001) only focus on business-stealing motive and ignore the welfare motive of R&D subsidization. This is because they do not consider any welfare analysis. Collie (2002) is among a few exceptions looking at welfare effect of subsidies but it addresses production subsidies rather than R&D subsidies. Spencer and Brander (1983) and Haaland and Kind (2008) are studies most closely related to our paper in terms of studying R&D subsidization, however, they restrict their attention to competition between a home firm and a foreign firm rather than that of two exporting firms as presented in our paper. Long et al. (2011), while studies the impact of trade liberalization on R&D, does not consider the subsidization issue. Similar to Neary and O’Sullivan (1999) and Leahy and Neary (2004), that paper looks at R&D cooperation/competition between firms rather than R&D competition/cooperation at the policy stage. To some extent, this paper is also related to Long and Staehler (2007) in terms of considering strategic behaviour of firms under different scenarios. Nevertheless, that paper focuses on public ownership and trade policy, not R&D investment and trade policy as our paper does.

The rest of the paper is structured as follows. Section 2 introduces a basic model of competition between exporting firms in an overseas market. In Section 3, exporting firms are additionally allowed to have domestic sales. For each case, the existence of an optimal R&D subsidy and its key characteristics are analyzed. The impacts of trade liberalization on firms’ output sales, cost-reducing R&D investments, productivity and social welfare are also examined. Section 4 concludes the paper by offering concluding remarks.
2 The model

Consider two domestic firms \( i \) and \( j \) whose products are entirely exported to a foreign country that does not produce these goods.\(^1\) The utility function of an overseas representative consumer is:

\[
u = \alpha q_i + \alpha q_j - \left( \frac{q_i^2}{2} + \frac{q_j^2}{2} + bq_i q_j \right), \ b \in [0, 1), \ \alpha > 0 \tag{1}\]

where \( q_i \) and \( q_j \) are consumption of the goods produced by the two firms respectively; \( b \) denotes the degree of substitution between the two goods (the higher the value of \( b \), the higher the degree of substitutability). When \( b = 0 \), the goods are completely independent and when \( b \) tends to its limit of 1, the goods are identical. This quadratic utility function is standard and has been used by Haaland and Kind (2008). For simplicity, assume the population size in the foreign market is equal to 1.

Let \( p_i \) and \( p_j \) denote the prices of the two goods in the foreign country. The consumer surplus of the foreign country can be expressed as:

\[
CS = u - p_i q_i - p_j q_j
\]

As the consumer maximizes his surplus with respect to the quantity of each good, the demand functions can be derived as the following:

\[
p_i = \alpha - (q_i + bq_j) \\
p_j = \alpha - (q_j + bq_i)
\]

Assume that the firms’ products are subject to a trade cost (e.g. import tariff, transportation or service cost) of rate \( \tau \) per unit of goods they export to the foreign market (\( \tau > 0 \)). By trade liberalization, it is meant a fall in \( \tau \). In the absence of R&D, firms face the same unit cost of production \( c \). These imply that in order to sell their products in the foreign market, firms have to bear the exporting cost of \( c + \tau \). To allow firms to be able to export even when no R&D activity is conducted, assume \( c + \tau < \alpha \). Firms invest in R&D to reduce their cost of production so that the cost of production after R&D is \( c - x_k \) where \( c \geq x_k \geq 0 \) (\( k = i, j \)) is the amount of R&D effort expended by firms. The R&D cost function \( r(x_k) \) takes the standard form with the following assumptions:

\(^1\)In this paper, the exporting country is referred to as the home country.
Assumption 1: The R&D cost function \( r(x_k) \):

- is positively valued: \( r(x_k) > 0 \), \( \forall x_k \geq 0 \);
- is strictly increasing: \( r'(x_k) > 0 \), \( \forall x_k > 0 \); and
- is strictly convex with curvature \( r''(x_k) > \frac{5}{4} \), \( \forall x_k > 0 \).

These assumptions, as will be shown later, are necessary for fulfilling sufficient conditions of maximization problems.\(^2\)

Also assume that the government helps each firm by providing an R&D subsidy of rate \( s_k (k = i, j) \) per unit of R&D investment. Hence, the profit function for a typical firm \( k (k = i, j) \) is:

\[
\pi_k = [p_k - (c - x_k) - \tau] q_k - r(x_k) + s_k x_k
\]

An assumption that is maintained throughout this paper is that firms obtain non-negative profits when they enter the production stage of the market. Each firm will maximize its profit while the domestic government will maximize total welfare. Because all goods are exported and not consumed in domestic market, domestic consumer surplus is zero. Hence, total welfare is equal to total firms’ profits less R&D subsidy costs:

\[
W = \sum_{k=i,j} \pi_k - \sum_{k=i,j} s_k x_k
\]

In this paper, we follow Long et al. (2011) in using Melitz (2003)’s definition of productivity. Here, firm productivity is the inverse of the marginal production cost:

\[
z_k = \frac{1}{c - x_k}, \ k = i, j
\]

and industry productivity is the inverse of the average marginal production cost:

\[
Z = \frac{\sum_{k=i,j} k}{\sum_{k=i,j} (c - x_k)}
\]

\(^2\)A typical example of such an R&D cost function is \( r(x_k) = Ax_k^2 + f \) where \( f \geq 0 \) is the fixed cost for setting up an R&D project and \( A > \frac{5}{8} \) is a constant.
The above setting provides us with a two-stage game. In the first stage, the government chooses how much to subsidize firms’ R&D efforts to maximize social welfare. In the second stage, the firms choose the R&D investment levels and export volumes to maximize their corresponding profits taking into account the R&D subsidy rate given in the first stage. We will solve this game using backward induction.

2.1 The pure monopolistic case

For a start, consider the case where firms are independent monopolies in their own market product lines. In other words, goods are imperfect substitutes so \( b = 0 \). Due to absence of competition, there will be no strategic interactions between the firms. Each firm will make its own decision on R&D investment and export sale regardless of the other’s.

The first order necessary conditions for a representative firm \( k \)’s profit maximization problem can be found by setting \( \frac{\partial \pi_k}{\partial q_k} = 0 \) and \( \frac{\partial \pi_k}{\partial x_k} = 0 \) \( (k = i, j) \) so that:

\[
(\alpha - c - \tau) + x_k - 2q_k = 0 \tag{6}
\]

\[
q_k + s_k - r'(x_k) = 0 \tag{7}
\]

As for the second order sufficient conditions, we have the following Hessian matrix:

\[
H = \begin{pmatrix}
-2 & 1 \\
1 & -r''(x_k)
\end{pmatrix}
\]

It can be seen that \( |H_1| = -2 < 0 \) and \( |H_2| = 2r''(x_k) - 1 > 0 \) as per Assumption 1. Thus, the second order sufficient conditions are satisfied for a maximum.

We now turn to the home government’s action. The government chooses R&D subsidy rates, \( s_k \) \( (k = i, j) \), to grant to firms in order to maximize welfare defined in (3). From (6) and (7) we obtain:
\[ r'(x_k) = \frac{\alpha - c - \tau}{2} + \frac{x_k}{2} + s_k \] 

(8)

\[ q_k = \frac{\alpha - c - \tau}{2} + \frac{x_k}{2} \] 

(9)

Substituting these into (3), the value of this social welfare is:

\[ W = \sum_{k=i,j} \left[ q_k^2 - r(x_k) \right] \]

\[ = \sum_{k=i,j} \left[ \frac{x_k^2}{4} + \frac{\alpha - c - \tau}{2} x_k + \frac{(\alpha - c - \tau)^2}{4} - r(x_k) \right] \]

This implies:

\[ \frac{\partial^2 W}{\partial x_k^2} = \frac{1}{2} x_k - r''(x_k) < 0, \forall k = i, j \]

or the function \( W \) is strictly concave. The maximizers of \( W \) can be established by setting \( \frac{\partial W}{\partial x_k} = 0 \) (\( k = i, j \)):

\[ \frac{\alpha - c - \tau}{2} + \frac{x_k}{2} - r'(x_k) = 0 \]

Comparing this with (8), we get:

\[ s_k = 0, \forall k = i, j \]

\[ q_k = r'(x_k), \forall k = i, j \]

From these results, it follows that:

**Proposition 1** When exporting firms are natural monopolies in their product lines in the foreign market, it is optimal for the government not to subsidize firms’ R&D activities.
Proposition 1 contains a striking result. Its intuition can be explained as follows. Facing with no competition at all, each exporting firm becomes a monopoly in its own market segment in the foreign market and enjoys the monopoly profits. Because the firm’s marginal export revenue and its marginal R&D spending cost cancel out each other, any firm’s extra profit will be equal to the value of R&D subsidy it receives from the government. Consequently, the government cannot use R&D subsidy to increase the exporting firms’ profit net of R&D subsidy cost for the welfare. This indicates that the optimal policy for the government is to withhold any R&D subsidy to the firms.

Regarding the impact of trade liberalization on the domestic economy, we can prove the following:

**Proposition 2** As soon as \( r'(c) > \frac{\alpha - \tau}{2} \), natural monopolistic exporting firms will conduct cost-reducing R&D and export goods to the foreign market. Trade liberalization in the foreign market: (i) leads to higher cost-reducing R&D investments of firms; (ii) raises their export sales; (iii) increases firms and industry productivity; and (iv) increases domestic welfare.

**Proof.** Given \( s_k = 0 \), from (6) and (7) we have:

\[
2r'(x_k) - x_k = \alpha - c - \tau
\]

Note that the right hand side (RHS) of this equation is a positive constant (because \( \alpha - c - \tau > 0 \)) while its left hand side (LHS) is an increasing function of \( x_k \) because its derivative with respect to \( x_k \) is \( 2r''(x_k) - 1 > 0 \). Given \( c \geq x_k \geq 0 \), the range of value of the LHS will be \([0; 2r'(c) - c]\). If \( 2r'(c) - c > \alpha - c - \tau \) or \( r'(c) > \frac{\alpha - \tau}{2} \), the equation yields a unique positive solution of \( x_k \) (the single crossing property). From this, the export volume can be calculated through the relation \( q_k = r'(x_k) \). It can be verified that with \( c \geq x_k \geq 0 \), the condition \( 0 \leq q_k \leq \alpha \) is automatically satisfied which guarantees that all the quantities and prices are non-negative (\( k = i, j \)).

From the above equation, it can be seen that a reduction in \( \tau \) will shift the graph of the RHS up resulting in an increase in \( x_k \). Because \( q_k \) is increasing in \( x_k \), it will also increase with the reduction in \( \tau \). Mathematically, we can write \( \frac{\partial q_k}{\partial \tau} < 0 \) and \( \frac{\partial q_k}{\partial \tau} < 0 \).

Due to symmetry, firms will have the same marginal production cost \((c - x_k)\). This implies that the industry marginal production cost defined in (??) is the same as the firms’ marginal production cost defined in (??):
\[ Z = z_k = \frac{1}{c-x_k} \]

Differentiating this productivity with respect to \( \tau \) gives:

\[
\frac{\partial Z}{\partial \tau} = \frac{\partial z_k}{\partial \tau} = \frac{1}{(c-x_k)^2} \cdot \frac{\partial x_k}{\partial \tau} < 0
\]

meaning the productivity increases when the trade cost falls.

Now, differentiating the social welfare with respect to \( \tau \) we get:

\[
\frac{\partial W}{\partial \tau} = 2 \left[ 2q_k \cdot \frac{\partial q_k}{\partial \tau} - r'(x_k) \cdot \frac{\partial x_k}{\partial \tau} \right] = -2q_k < 0
\]

because \(-1 + \frac{\partial x_k}{\partial \tau} = \frac{2q_k}{\partial \tau}\) based on (6) and \(q_k - r'(x_k) = 0\) as previously derived. This implies domestic welfare is improved with a fall in the trade cost.

The results contained in this proposition are quite intuitive. A lower \( \tau \) allows firms to expand their export sales and reap more profits even when R&D spending is held fixed. This export sale expansion raises the marginal benefits of undertaking cost-reducing R&D so firms increase their R&D spending. Both these actions raise firms’ profits even further. With more R&D spending, marginal cost of production decreases implies an enhancement of productivity at both firms’ and industry level. Since firms’ profits are higher, domestic welfare is also higher because domestic welfare, in the absence of subsidy and consumer surplus, is equal to the sum of firms’ profits. Similar to Long et al. (2011), we refer to this as the direct effect of trade liberalization.

### 2.2 The duopolistic competition

Now goods are close substitutes \((0 < b < 1)\) and firms compete in a Cournot game. Conditional on the government’s decision made regarding R&D subsidy in the first stage, each firm chooses how much to invest in R&D and how much to export to maximize its profit defined in (2). The first order necessary conditions for firm \(i\)'s profit maximization problem give:

\[
(\alpha - c - \tau) + x_i - bq_j - 2q_i = 0 \tag{10}
\]

\[
q_i + s_i - r'(x_i) = 0 \tag{11}
\]
and similar for firm $j$. The Hessian matrix of the second order sufficient conditions for firm $i$ is:

$$H = \begin{pmatrix} -2 & 1 \\ 1 & -r''(x_i) \end{pmatrix}$$

and similar for firm $j$. It can be seen that $|H_1| = -2 < 0$ and $|H_2| = 2r''(x_i) - 1 > 0$ according to Assumption 1. Hence, the second order sufficient conditions are satisfied for a maximum.

In the first stage, the government, having known the firms’ strategic response functions in (10) and (11), chooses R&D subsidy rates $(s_i, s_j)$ to grant to firms in order to maximize the social welfare defined in (3) which can now be rewritten as:

$$W = q_i^2 - r(x_i) + q_j^2 - r(x_j)$$

Setting $\frac{\partial W}{\partial s_i} = 0$ and $\frac{\partial W}{\partial s_j} = 0$ yields the following:

$$2q_i \cdot \frac{\partial q_i}{\partial s_i} - r'(x_i) \cdot \frac{\partial x_i}{\partial s_i} + 2q_j \cdot \frac{\partial q_j}{\partial s_i} - r'(x_j) \cdot \frac{\partial x_j}{\partial s_i} = 0$$

$$2q_i \cdot \frac{\partial q_i}{\partial s_j} - r'(x_i) \cdot \frac{\partial x_i}{\partial s_j} + 2q_j \cdot \frac{\partial q_j}{\partial s_j} - r'(x_j) \cdot \frac{\partial x_j}{\partial s_j} = 0$$

where $q_i$ and $x_i$ (and, similarly, $q_j$ and $x_j$) are given in (10) and (11). After some tedious calculations, these equations can be simplified as:

$$\left[ 2q_i - bq_j + \frac{(4-b^2)s_i}{b} \right] \frac{\partial q_i}{\partial s_i} - \left[ q_i + s_i + \frac{2s_j}{b} \right] \frac{\partial x_i}{\partial s_i} = 0$$

$$\left[ 2q_j - bq_i + \frac{(4-b^2)s_j}{b} \right] \frac{\partial q_j}{\partial s_j} - \left[ q_j + s_j + \frac{2s_i}{b} \right] \frac{\partial x_j}{\partial s_j} = 0$$

It can be seen that the first order conditions yield a symmetric outcome at which $s_i = s_j = s$, $q_i = q_j = q$, and $x_i = x_j = x$. From (10) and (11), the following is obtained:

$$q = \frac{\alpha - e - \tau + x}{b + 2}$$

Using this result to recalculate the social welfare we have:
\[ W = 2 \left[ q^2 - r(x) \right] = 2 \left[ \left( \frac{\alpha - c - \tau + x}{b + 2} \right)^2 - r(x) \right] \]

which means:

\[ \frac{\partial^2 W}{\partial x^2} = 2 \left[ \frac{-2}{(b+2)^2} - r''(x) \right] \]

The function \( W \) is strictly concave since \( r''(x) \cdot (b + 2)^2 > 2, \forall b \in (0, 1) \). Differentiating this welfare function with respect to \( s \) and setting it to zero gives:

\[ q = \frac{- (b + 2)s}{b} \quad (12) \]

Inserting the result in (12) into (10) and (11) under symmetry delivers:

\[ x = \frac{-(b + 2)^2s}{b} - (\alpha - c - \tau) \quad (13) \]

Because export volume and R&D investment are non-negative, we must have \( s < 0 \). Let \( \theta = -s > 0 \). The condition that must be met by \( \theta \) is that

\[ \frac{b(\alpha - \tau)}{(b + 2)^2} \geq \theta \geq \frac{b(\alpha - c - \tau)}{(b + 2)^2} \]

so that \( c \geq x \geq 0 \). It can be verified that this range of value for \( \theta \) also guarantees that \( p = \alpha - (b + 1)q \geq 0 \) and \( q > 0 \).

Substituting the obtained results into (11) gives:

\[ \frac{2\theta}{b} - r'(x) = 0 \quad (14) \]

From this equation, we can now state:

**Proposition 3** When goods are imperfect substitutes and exporting firms only compete in an overseas market, if \( r'(c) > \frac{2(\alpha - \tau)}{(b + 2)^2} \), the social optimum can be achieved as a Nash equilibrium by applying an optimal R&D tax. This optimal R&D tax is increasing in the trade cost but decreasing in the degree of substitutability between goods.
Proof. We will prove this proposition in two parts. In the first part, we prove the existence of a unique and negative value of $s$ (i.e. an optimal R&D tax). In the second part, we prove that $s$ is increasing in $\tau$ but decreasing in $b$.

To prove the first part, we consider the LHS of (14) which is a function of $\theta$: $f(\theta) = \frac{2\theta}{b} - r'(x)$. Differentiating this function with respect to $\theta$ yields:

$$f'(\theta) = \frac{2}{b} - r''(x) \cdot \frac{\partial x}{\partial \theta} = \frac{2}{b} - r''(x) \cdot \frac{(b+2)^2}{b}.$$

Because $r''(x) \cdot \frac{(b+2)^2}{b} > \frac{2}{b}$ according to Assumption 1, $f'(\theta) < 0$ meaning the LHS of (14) is a decreasing function of $\theta$ while its LHS is a constant (equal to zero). At $\theta = \frac{b(\alpha-c-\tau)}{(b+2)^2}$, $f(\theta) = 0$. When $\theta = \frac{b(\alpha-\tau)}{(b+2)^2}$, $f(\theta) = \frac{2(\alpha-\tau)}{(b+2)^2} - r'(c) < 0$ because $r'(c) > 2(\alpha-\tau)(b+2)^2$ as per our above stated assumption. Hence, $\theta = \frac{b(\alpha-c-\tau)}{(b+2)^2}$ is the unique value that solves (14). Therefore, $s = -\theta = -\frac{b(\alpha-c-\tau)}{(b+2)^2} < 0$ is the unique optimal R&D tax that should be applied by the government to firms’ R&D efforts.

Now, differentiating $s$ with respect to $\tau$ we get:

$$\frac{\partial s}{\partial \tau} = \frac{b}{(b+2)^2} > 0$$

This implies that $s$ is increasing in $\tau$. Hence, a decrease in $\tau$ leads to a decrease in $s$ (s becomes more negative or a higher level of R&D tax should be levied).

Similarly, differentiating $s$ with respect to $b$:

$$\frac{\partial s}{\partial b} = -(\alpha - c - \tau) \cdot \frac{4-b^2}{(b+2)^4} < 0$$

This reflects that an increase in $b$ entails a decrease in $s$.

This proposition contains three results. The first result is quite interesting. Unlike the case of monopolies, zero subsidy is no longer optimal. More surprisingly, the socially optimal policy turns out to be the government taxing R&D activity instead of subsidizing it. This can be explained on the following ground. When exporting firms from a country compete with each other in a foreign market, their competition may make the home country as a whole fail to exploit its potential monopoly power in the foreign market. Too much R&D conducted will lead to over-production for the two domestic exporting firms. To avoid this situation, the home government should impose an R&D
tax, at the same rate, on both firms. This optimal R&D tax guarantees that social welfare will be maximized and firms will have no incentive to do less or more R&D and, hence, to produce less or more exported products.

The second result says that when there is a reduction in the trade cost, the optimal action of the home government is to tax the firms’ R&D investments more heavily. This is because lower trade cost expands firms’ export volumes and thus raises firms’ willingness to invest in cost-reducing R&D. To reduce firms’ excessive R&D spending so that over-production, which erodes the home country’s monopoly power in the foreign market, can be avoided, the government needs to raise the R&D tax rate. This action will result in an improvement in social welfare because firms obtain more profits from exports (even though no more R&D investments occur) and the government collects more taxes.

The last result is concerned about the effect of a change in $b$ on the optimal R&D subsidy. Given that $b$ captures the degree of substitutability between goods, an increase in $b$ means the goods become less differentiated and, thus, firms face with fiercer competition from each other.\footnote{According to Haaaland and Kind (2008), an increase in $b$ implies a decrease in market demand. In other words, the size of the market gets smaller when goods become less differentiated.} Due to higher competition pressure, firms tend to act more aggressively. To further prevent firms from undertaking excessive R&D and production, a higher R&D tax is required.

We now examine the economic impact of trade liberalization on the home country. To derive the comparative static effects of a reduction in $\tau$, we totally differentiate the above obtained equilibrium condition. The results can be summarized in the proposition below:

**Proposition 4** When exporting firms only compete in a foreign market and 
\[
\frac{r'(c)}{\frac{2(a-\tau)}{(b+2)^2}} > \frac{a-\tau}{(b+2)^2},
\]
trade liberalization in that market has no impact on firms’ cost-reducing R&D spending, their productivity and industry productivity. However, it still raises firms’ export sales and enhances domestic welfare.

**Proof.** The proof of this proposition is quite straightforward. Indeed, making use of (12) and (13), we get:

\[
\frac{\partial x}{\partial \tau} = -\frac{(b+2)^2}{b} \cdot \frac{\partial s}{\partial \tau} + 1 = -\frac{(b+2)^2}{b} \cdot \frac{b}{(b+2)\tau} + 1 = 0
\]
\frac{\partial q}{\partial \tau} = - \frac{(b+2)}{b} \frac{\partial q}{\partial \tau} = - \frac{1}{b+2} < 0

Due to symmetry, in equilibrium, firms’ and industry productivities are the same \( Z = z = \frac{1}{c-x} \). Differentiating this with respect to \( \tau \) delivers:

\frac{\partial Z}{\partial \tau} = \frac{\partial z}{\partial \tau} = \frac{1}{(c-x)^2} \frac{\partial x}{\partial \tau} = 0

The effect on welfare is:

\frac{\partial W}{\partial \tau} = 2 \left[ 2q \frac{\partial q}{\partial \tau} - r'(x) \frac{\partial x}{\partial \tau} \right] = - \frac{4q}{b+2} < 0

All these imply that \( x \) and \( Z \) are independent from \( \tau \). By contrast, \( W \) and \( q \) are decreasing in \( \tau \). A fall in \( \tau \) will increase both \( W \) and \( q \).

Unlike the case of independent monopolies, here, trade liberalization has no impact on firms’ R&D spending and productivity. This is because trade liberalization now has two different effects: a direct effect and an indirect effect. The direct effect, as explained under Proposition 2, encourages firms to conduct more cost-reducing R&D. By contrast, the indirect effect dampens firms’ R&D efforts through triggering an increase in the R&D tax rate. At the optimal (in terms of welfare), these two opposing effects exactly cancel out each other so there is no change in firms’ R&D spending. Because there is no improvement made on firms’ marginal production cost, the firms’ and industry productivity stay unchanged.

Although there is no change in R&D spending, trade liberalization still increases firms’ export sales because the trade cost is lower making the whole exporting cost lower. This sale expansion allows firms to enjoy higher profits. In the meantime, the government gets more revenue through higher R&D taxation. All this leads to a higher level of domestic welfare.

3 Adding domestic sales

In addition to the competition in the foreign market as described in Section 2, we now further assume that competition between two exporting firms also takes place in the home market. As there are now two markets, we need to make some small changes in notation. Define the home market as Country 1 and the foreign market as Country 2. Assume the population size in each country is equal to 1 and consumers everywhere have the same preferences.
for simplicity. The representative consumer in home country derives utility from consuming goods supplied by firms:

\[ u_i = \alpha q_{i1} + \alpha q_{j1} - \left( \frac{q_{i1}^2}{2} + \frac{q_{j1}^2}{2} + b q_{i1} q_{j1} \right), \quad b \in [0, 1), \quad \alpha > 0 \quad (15) \]

and similar for the consumer in the foreign country. Here, \( q_{i1} \) and \( q_{j1} \) denote the consumption of goods produced by the firms. The first subscript is used to indicate the firm producing the consumption good and the second subscript refers to the country of consumption. The domestic consumer surplus is:

\[ CS_1 = u_i - p_{i1} q_{i1} - p_{j1} q_{j1} \]

From this, the inverse demand functions are:

\[ p_{i1} = \alpha - (q_{i1} + bq_{j1}) \]
\[ p_{j1} = \alpha - (q_{j1} + bq_{i1}) \]

Using these results, the maximized domestic consumer surplus can be calculated as:

\[ CS_1 = \frac{1}{2} (q_{i1}^2 + q_{j1}^2) + bq_{i1} q_{j1} \]

The inverse demand functions for goods in the overseas market are the same as previously described in Section 2. Hence, the profit function for firm \( i \) is:

\[ \pi_i = [p_{i1} - (c - x_{i1})] q_{i1} + [p_{i2} - (c - x_{i2}) - \tau] q_{i2} - r(x_i) + s_i x_i \quad (16) \]

and similar for firm \( j \). In this profit function, the first two terms capture the firm’s domestic sale revenue and export sale revenue respectively while the last two terms are R&D investment spending and support from the government.

Welfare of the home country will be:

\[ W = \pi_i + \pi_j + CS_1 - s_i x_i - s_j x_j \quad (17) \]

A slight difference between this welfare function and the one defined in Section 2 is the inclusion of consumer surplus. Any R&D policies should now also take this component into account.
3.1 The pure monopolistic case

As before, when \( b = 0 \), goods are imperfect substitutes and each firm is a pure monopoly in its own market segment. A representative firm \( k \) \((k = i, j)\) chooses R&D spending, \( x_k \), domestic sale, \( q_{k1} \), and export volume, \( q_{k2} \), to maximize its profit. The first order necessary conditions deliver:

\[
(\alpha - c) + x_k - 2q_{k1} = 0 \tag{18}
\]

\[
(\alpha - c - \tau) + x_k - 2q_{k2} = 0 \tag{19}
\]

\[
q_{k1} + q_{k2} + s_k - r'(x_k) = 0 \tag{20}
\]

The corresponding Hessian matrix for the second order conditions is:

\[
H = \begin{pmatrix}
-2 & 0 & 1 \\
0 & -2 & 1 \\
1 & 1 & -r''(x_k)
\end{pmatrix}
\]

and similar for firm \( j \). It can be seen that \( |H_1| = -2 < 0 \), \(|H_2| = 4 > 0\), and \( |H_3| = 4[1 - r''(x_k)] \) < 0 under Assumption 1. Therefore, the second order conditions are met for a maximum.

In the first stage, the social welfare given in (17) can be rewritten as:

\[
W = \frac{3q_{i1}^2}{2} + q_{i2}^2 - r(x_i) + \frac{3q_{j1}^2}{2} + q_{j2}^2 - r(x_j)
\]

The government’s choice of \( s_k \) to maximize aggregate welfare is pin down by setting \( \frac{\partial W}{\partial s_k} = 0 \) \((k = i, j)\) which implies:

\[
3q_{k1} \cdot \frac{\partial q_{k1}}{\partial s_k} + 2q_{k2} \cdot \frac{\partial q_{k2}}{\partial s_k} - r'(x_k) \cdot \frac{\partial x_k}{\partial s_k} = 0
\]

where \( q_{k1}, q_{k2}, \) and \( x_k \) are determined by the system of above first order conditions.

The second order sufficient conditions \((k = i, j)\) are:
\[
\frac{\partial^2 W}{\partial s_k^2} = 3 \left( \frac{\partial q_{k1}}{\partial s_k} \right)^2 + 3 q_{k1} \cdot \frac{\partial^2 q_{k1}}{\partial s_k^2} + 2 \left( \frac{\partial q_{k2}}{\partial s_k} \right)^2 + 2 q_{k2} \cdot \frac{\partial^2 q_{k2}}{\partial s_k^2} \\
- r''(x_k) \cdot \left( \frac{\partial x_k}{\partial s_k} \right)^2 - r'(x_k) \cdot \frac{\partial^2 x_k}{\partial s_k}
\]

From (18) - (20) we have:

\[
\frac{\partial q_{k1}}{\partial s_k} = \frac{\partial q_{k2}}{\partial s_k} = \frac{1}{2} \cdot \frac{\partial x_k}{\partial s_k}
\]

\[
\frac{\partial q_{k1}}{\partial s_k} + \frac{\partial q_{k2}}{\partial s_k} + 1 = r''(x_k) \cdot \frac{\partial x_k}{\partial s_k}
\]

These two equations together deliver:

\[
[r''(x_k) - 1] \cdot \frac{\partial x_k}{\partial s_k} = 1
\]

This implies \( \frac{\partial x_k}{\partial s_k} \neq 0 \). Substituting the above results into the first order condition for welfare maximization and rearranging gives:

\[
\left( \frac{q_{k1}}{2} - s_k \right) \cdot \frac{\partial x_k}{\partial s_k} = 0
\]

This points out that:

\[
q_{k1} = 2 s_k
\]

Consequently, from (18) and (19):

\[
x_k = 4 s_k - (\alpha - c)
\]

\[
q_{k2} = 2 s_k - \frac{\tau}{2}
\]
Using these results, we can work out $\frac{\partial^2 W}{\partial s_k^2} = -80 < 0$. Hence, the second order sufficient conditions are satisfied for a maximum. The range of $s_k$ is determined through the set of conditions which make quantities and prices positive. In particular, by setting $0 \leq q_k \leq \alpha$ and $0 \leq x_k \leq c$, it is required that $\frac{\alpha-c}{4} \leq s_k \leq \frac{\alpha}{4}$.

We can solve for the optimal value of $s_k$ based on (20) which, after rearranging, becomes:

$$5s_k - \frac{\tau}{2} - r'(x_k) = 0 \quad \text{(24)}$$

This leads to the following proposition:

**Proposition 5** If $r'(c) \geq \frac{5\alpha-2\tau}{4}$, the welfare maximizing R&D subsidy expended by the government to each monopoly firm, whose products are sold in both home and foreign markets, exists and is uniquely determined. In addition, this subsidy rate is positive and decreasing in the trade cost.

**Proof.** Denoting the LHS of the above equation as $g(s_k) = 5s_k - \frac{\tau}{2} - r'(x_k)$ and differentiating it with respect to $s_k$ we get:

$$g'(s_k) = 5 - r''(x_k) \frac{\partial x_k}{\partial s_k} = 5 - 4r''(x_k)$$

According to Assumption 1, $g'(s_k) < 0$ or $g(s_k)$ is decreasing in $s_k$. Given $\frac{\alpha-c}{4} \leq s_k \leq \frac{\alpha}{4}$, the range of value of $g(s_k)$ is $\left[\frac{5\alpha-2\tau}{4} - r'(c); \frac{5\alpha-(c-2\tau)}{4}\right]$. Clearly, $\frac{5(\alpha-c)-2\tau}{4} > 0$. If $\frac{5\alpha-2\tau}{4} - r'(c) \leq 0$ or $r'(c) \geq \frac{5\alpha-2\tau}{4}$, the equation earns a unique and positive value of $s_k$.

From (24), an increase in $\tau$ will shift the graph of the LHS down for every $s_k$. The shock will result in a lower equilibrium value of $s_k$. In other words, $s_k$ is decreasing in $\tau$.

The main difference relative to the case of no domestic sales investigated in Sub-section 2.1 is that the government should really subsidize monopolistic firms. This is because when firms conduct business in the home market, consumer surplus is part of the social welfare function. Any change in firms’ production behaviours not only affect firms’ profits but also consumer surplus. Through subsidizing firms’ R&D activities, the government boosts firms’ exports as well as their domestic sales because firms (and the entire domestic
economy) become more efficient. Although this policy does not raise firms’ profits net of subsidy cost because firms’ marginal revenue is cancelled out by marginal cost (this is similar to what is discussed under Proposition 1), it increases consumer surplus. This will lead to a higher level of welfare. To see that the government has an incentive to grant R&D subsidy to increase domestic consumer surplus, we take the following derivative:

$$\frac{\partial C_s}{\partial s} = q_{i1}, \frac{\partial q_{11}}{\partial s} = 2q_{i1} > 0$$

Consumer surplus increases with cost-reducing R&D expenditure because the monopolies charge a lower price when marginal cost of production is lower. Because firms do not take this effect into account when making decision on R&D spending, the government should provide an R&D subsidy.

It should be noted that the optimal R&D subsidy is decreasing in the trade cost. Put it the other way, trade liberalization, which reduces trade cost, results in an increase in the optimal R&D subsidy. A fall in the trade cost raises firms’ export sales and make the marginal benefit of undertaking cost-reducing R&D become higher. By increasing R&D subsidy, the government creates more incentive for firms to do R&D and expand their production and sales (both at home and overseas). As the extra benefits from this policy action (i.e. higher consumer surplus, higher firms’ revenues) outweights its costs (R&D spending plus R&D subsidy), the policy is welfare enhancing.

Regarding the comparative statics of trade liberalization, we arrive at:

**Proposition 6** When firms are natural monopolies in their own product lines in both home and foreign markets and $r'(e) \geq \frac{5a-2\tau}{4}$, trade liberalization in the foreign market: (i) increases a firm’s R&D spending; (ii) increases the firm’s export volume, its domestic sales and, hence, its overall sales; (iii) improves firms’ and industry productivity; and (iv) raises domestic welfare.

**Proof.** As shown above, $s_k$ is decreasing in $\tau$. Mathematically, it means $\frac{\partial s_k}{\partial \tau} < 0$. Therefore, we can derive:

$$\frac{\partial q_{k1}}{\partial \tau} = \frac{2\partial s_k}{\partial \tau} < 0$$

$$\frac{\partial q_{k2}}{\partial \tau} = \frac{2\partial s_k}{\partial \tau} - \frac{1}{2} < 0$$

$$\frac{\partial x_k}{\partial \tau} = \frac{4\partial s_k}{\partial \tau} < 0$$

20
\[ \frac{\partial q_k}{\partial \tau} = 4\frac{\partial s_k}{\partial \tau} - \frac{1}{2} < 0 \]

where in the last equation we denote \( q_k = q_{k1} + q_{k2} \) as the firm’s overall sales.

In equilibrium, firms’ productivity is the same as industry productivity \( Z = z_k = \frac{1}{c-x_k} \). Differentiating this with respect to \( \tau \) gives:

\[ \frac{\partial Z}{\partial \tau} = \frac{\partial z_k}{\partial \tau} = \frac{1}{(c-x_k)^2} \frac{\partial x_k}{\partial \tau} < 0 \]

As for the welfare effect, we have:

\[
\frac{\partial W}{\partial \tau} = 2 \left[ \frac{3q_{k1}}{\partial \tau} \frac{\partial q_{k1}}{\partial \tau} + 2q_{k2} \frac{\partial q_{k2}}{\partial \tau} - r'(x_k) \frac{\partial x_k}{\partial \tau} \right] \\
= 2 \left[ 12s_k \frac{\partial s_k}{\partial \tau} + (4s_k - \tau) \left( \frac{2\partial s_k}{\partial \tau} - \frac{1}{2} \right) - \left( 5s_k - \frac{\tau}{2} \right) \right] \frac{4\partial s_k}{\partial \tau} \\
= -(4s_k - \tau)
\]

Given that \( \frac{\alpha - c}{4} \leq s_k \leq \frac{\alpha}{4} \) then \( 4s_k - \tau \geq \alpha - c - \tau > 0 \). Therefore, \( \frac{\partial W}{\partial \tau} < 0 \).

As previously explained under Proposition 4, trade liberalization affects a representative firm’s export and R&D spending both directly and indirectly. In this case, because an R&D subsidy is put in place (instead of a zero subsidy as in Sub-section 2.1 and a tax as in Sub-section 2.2), the two effects, direct and indirect, strengthen each other. As the result, the net outcome of this shock is an increase in R&D investments and a further expansion of export sales. The increase in R&D spending reduces the marginal cost of production and, thus, raises productivity both at the firm and industry level. Domestic sales will also be higher because the firm becomes more efficient. In total, the firm’s overall sales are higher.

Since output increases with trade liberalization, it follows that firms’ profits and consumer surplus will rise more than enough to compensate for the subsidy cost. This implies an improvement in welfare.

### 3.2 The duopolistic case

In this case \( b \neq 0 \). The first order conditions from firm \( i \)'s profit maximization are:

\[ (\alpha - c) + x_i - bq_{j1} - 2q_{i1} = 0 \quad (25) \]
These equations can be simplified as follows:
\[(\alpha - c - \tau) + x_i - bq_{j2} - 2q_{j2} = 0 \quad (26)\]

\[q_{i1} + q_{i2} + s_i - r'(x_i) = 0 \quad (27)\]

and similar for firm \(j\). The Hessian matrix of second order conditions are:

\[H = \begin{pmatrix} -2 & 0 & 1 \\ 0 & -2 & 1 \\ 1 & 1 & -r'(x_i) \end{pmatrix} \]

We have \(|H_1| = -2 < 0\), \(|H_2| = 4 > 0\), and \(|H_3| = 4 [1 - r''(x_i)] < 0\) meaning the second order conditions are satisfied for a maximum.

In the first stage, the aggregate welfare is:

\[W = \frac{3q_{i1}^3}{2} + q_{i2}^2 - r(x_i) + \frac{3q_{j1}^3}{2} + q_{j2}^2 - r(x_j) + bq_{i1}q_{j1} \]

The government’s welfare maximization delivers the first order conditions:

\[3q_{i1} \cdot \frac{\partial q_{i1}}{\partial s_i} + 2q_{i2} \cdot \frac{\partial q_{i2}}{\partial s_i} - r'(x_i) \cdot \frac{\partial x_i}{\partial s_i} + 3q_{j1} \cdot \frac{\partial q_{j1}}{\partial s_i} + 2q_{j2} \cdot \frac{\partial q_{j2}}{\partial s_i} - r'(x_j) \cdot \frac{\partial x_j}{\partial s_j} + bq_{i1} \cdot \frac{\partial q_{i1}}{\partial s_j} + bq_{j1} \cdot \frac{\partial q_{j1}}{\partial s_j} = 0 \]

where \(q_{i1}, q_{i2}, \) and \(x_i\) (and similar for \(q_{j1}, q_{j2}, \) and \(x_j\)) are given in (25) - (27).

These equations can be simplified as follows:

\[\left[ q_{i1} - \frac{2q_{i1}}{b} - \frac{(b^2 - 4)b}{b} - \frac{(b^2 - 4)s_i}{b} \right] \cdot \frac{\partial q_{i1}}{\partial s_i} + \left( 2q_{i2} - \frac{4q_{i2}}{b} \right) \cdot \frac{\partial q_{i2}}{\partial s_i} + \left( \frac{q_{i1}}{b} - q_{i2} - s_i - \frac{2s_i}{b} \right) \cdot \frac{\partial x_i}{\partial s_i} = 0 \]

\[\left[ q_{j1} - \frac{2q_{j1}}{b} - \frac{(b^2 - 4)b}{b} - \frac{(b^2 - 4)s_i}{b} \right] \cdot \frac{\partial q_{j1}}{\partial s_j} + \left( 2q_{j2} - \frac{4q_{j2}}{b} \right) \cdot \frac{\partial q_{j2}}{\partial s_j} + \left( \frac{q_{j1}}{b} - q_{j2} - s_j - \frac{2s_j}{b} \right) \cdot \frac{\partial x_j}{\partial s_j} = 0 \]

In addition, we can also work out that \(\frac{\partial q_{i1}}{\partial s_i} = \frac{\partial q_{i2}}{\partial s_i} = \frac{\partial q_{j1}}{\partial s_j} = \frac{\partial q_{j2}}{\partial s_j}\), and \(\frac{\partial q_{i1}}{\partial s_j} = \frac{\partial q_{i2}}{\partial s_j}\) which allow us to simplify the above equations further:
These first order conditions imply a symetric outcome where \( s_i = s_j = s,\)
\( q_{i1} = q_{j1} = q_1,\) \( q_{i2} = q_{j2} = q_2,\) and \( x_i = x_j = x.\) Recalculating the social welfare we get:
\[
W = (b + 3)q_1^2 + 2q_2^2 - 2r(x)
\]
which in turn imply the following after rederiving the first order condition:
\[
\frac{q_1}{b} - q_2 - \frac{(b+2)s}{b} = 0
\]
Using this result, we can figure out:
\[
q_1 = \frac{(b + 2)^2 s - b\tau}{(b + 2)(1 - b)} \quad \text{(28)}
\]
\[
q_2 = \frac{(b + 2)^2 s - \tau}{(b + 2)(1 - b)} \quad \text{(29)}
\]
\[
x = \frac{(b + 2)^2 s - b\tau - (1 - b)(\alpha - c)}{(1 - b)} \quad \text{(30)}
\]
Now, we check for the second order condition:
\[
\frac{\partial^2 W}{\partial s^2} = \frac{2(b+2)^2}{(1-b)^2} \left[ b + 5 - r''(x)(b + 2)^2 \right]
\]
It is easy to check that \( \max_{b\in[0,1]} \frac{b+5}{(b+2)^2} = \frac{5}{4}.\) From Assumption 1, \( \frac{\partial^2 W}{\partial s^2} < 0 \)
implying that the second order condition is satisfied for a maximum.
To make sure that quantities and prices are non-negative, we need to impose that \( 0 \leq x \leq c,\) and \( 0 \leq (b + 1)q_1 \leq \alpha,\) as well as \( 0 \leq (b + 1)q_2 \leq \alpha.\) These lead to the following:
\[
\frac{(1 - b)(\alpha - c)}{(b + 2)^2} + \frac{b\tau}{(b + 2)^2} \leq s \leq \frac{(1 - b)\alpha}{(b + 2)^2} + \frac{b\tau}{(b + 2)^2}
\]

(31)

Given this setting and conditions, we can derive the following:

**Proposition 7** When firms compete in both home and foreign markets, if \( r'(c) \geq \frac{(b+5)\alpha-2\tau}{(b+2)^2} \), the welfare maximizing R&D subsidy expended by the government to each firm exists, is positively valued and uniquely determined. While the monotonicity of this optimal R&D subsidy in the trade cost depends on the curvature of the R&D cost function, it is unambiguously decreasing in the degree of substitutability between the goods.

**Proof.** Substituting results in (28) and (29) into (27) and rearranging gives:

\[
\frac{(b + 5)s}{1 - b} - \frac{(b + 1)\tau}{(b + 2)(1 - b)} - r'(x) = 0
\]

(32)

Define \( h(s) = \frac{(b+5)s}{1-b} - \frac{(b+1)\tau}{(b+2)(1-b)} - r'(x) \). We have:

\[
h'(s) = \frac{b+5}{1-b} - r''(x) \frac{\partial_s}{\partial s} = \frac{b+5}{1-b} - r''(x) \frac{(b+2)^2}{1-b}
\]

As \( r''(x) \frac{(b+2)^2}{1-b} > b + 5 \), the numerator is positive so \( \frac{\partial s}{\partial \tau} < 0 \) or \( s \) is decreasing in \( \tau \). By contrast, if \( r''(x) \frac{(b+2)^2}{1-b} < b + 5 \), the numerator is negative so \( \frac{\partial s}{\partial \tau} > 0 \) or \( s \) is increasing in \( \tau \).

When \( r''(x) \frac{(b+2)^2}{1-b} = b + 5 \), \( \frac{\partial s}{\partial \tau} = 0 \) implying that \( s \) is unaffected by a change in \( \tau \).

Differentiating both sides of (32) with respect to \( \tau \) and rearranging gives:

\[
\frac{\partial s}{\partial \tau} = \frac{(b + 1) - b( b + 2).r''(x)}{(b + 2) \left[(b + 5) - (b + 2)^2r''(x)\right]}
\]

(33)

It should be noted that the denominator is always negative. If \( r''(x) < \frac{b+1}{(b+2)^2} \), the numerator is positive so \( \frac{\partial s}{\partial \tau} < 0 \) or \( s \) is decreasing in \( \tau \). By contrast, if \( r''(x) > \frac{b+1}{(b+2)^2} \), the numerator is negative so \( \frac{\partial s}{\partial \tau} > 0 \) or \( s \) is increasing in \( \tau \).

Similarly, differentiating (32) with respect to \( b \) and simplifying we obtain:
\[
[(b + 5) - r''(x). (b + 2)^2] \cdot \frac{s}{b} = \frac{r''(x)(b+2)(4-b)-6}{1-b} \cdot s - \frac{r''(x)(b+2)^2 - (b^2+2b+3)}{(b+2)^2(1-b)} \cdot r
\]

Given the range of values of \( s \) in (31), after some tedious computation, it can be verified that the RHS of the above equation is positive. Because \((b + 5) - r''(x). (b + 2)^2 < 0\) then \(\frac{\partial s}{\partial b} < 0\) meaning \(s\) is decreasing in \(b\).

Unlike the results obtained under Proposition 3 where an R&D tax should be imposed, with firms also trading in the homemarket, the government’s optimal policy is to subsidize R&D. This is very much because of the consumer surplus motive. In this case, the gain in consumer surplus due to R&D subsidy, which lowers the product prices by lowering firms’ marginal production cost, is more than sufficient to compensate for the associated costs so the government has an incentive to grant R&D subsidy. To this extent, the results obtained here are similar to those under Proposition 5.

The main difference relative to the previous cases is that the effect of trade liberalization on optimal R&D subsidy is no longer monotonic. In particular, it is dependent on the curvature of the R&D cost function. As we know, when trade liberalization occurs, firms enjoy more profits even if R&D spending is held fixed. If the R&D cost function is highly convex (R&D is a very costly activity), holding R&D investments fixed or even a slight decrease in R&D efforts will allow firms to save a great deal of R&D spending. In terms of welfare, the society will be better off if firms do not change or conduct less R&D because the savings (of R&D spending and R&D subsidy) obtained from doing so more than outweighs any reduction in firms’ profits and/or consumer surplus. To discourage firms from doing any further R&D, the government reduces its R&D subsidy extended to firms. However, when the R&D cost function is not so convex, the marginal benefit from implementing an R&D project is greater than its corresponding cost, the government should encourage firms to do more R&D by increasing the R&D subsidy level in the face of trade liberalization.

As discussed under Proposition 3, the total market demand (in this case, both at home and overseas) tends to decrease in \(b\). When \(b\) increases, the demand for goods is low, the gain in consumer surplus, due to the drop in prices, is not enough to offset the fall in firms’ profits obtained from both home and foreign market so the government has an incentive to cut down the subsidy level.

As for the impacts of trade liberalization on the home economy, we can show that:
**Proposition 8** When firms compete in both home and foreign markets and \( r'(c) \geq \frac{(b+5)\alpha-2\tau}{(b+2)^2} \), trade liberalization in the foreign market: (i) increases a firm’s R&D spending; (ii) increases the firm’s export volumes, its domestic sales and, hence, its total sales; (iii) improves the firm’s and industry productivity; and (iv) raises social welfare.

**Proof.** Using (28) - (30) and then (33), we obtain the following partial derivatives:

\[
\frac{\partial q_1}{\partial \tau} = \frac{2}{(b+2)[(b+5)-(b+2)^2r''(x)]} < 0
\]

\[
\frac{\partial q_2}{\partial \tau} = \frac{(b+2)^2r''(x)-(b+3)}{(b+2)[(b+5)-(b+2)^2r''(x)]} < 0
\]

\[
\frac{\partial x}{\partial \tau} = \frac{2}{[(b+5)-(b+2)^2r''(x)]]} < 0
\]

Defining \( q = q_1 + q_2 \) as a firm’s total sales then:

\[
\frac{\partial q}{\partial \tau} = \frac{\partial q_1}{\partial \tau} + \frac{\partial q_2}{\partial \tau} < 0
\]

The industry productivity is equal to firm’s productivity \( Z = z = \frac{1}{c-x} \). Differentiating with respect to \( \tau \) delivers:

\[
\frac{\partial Z}{\partial \tau} = \frac{\partial z}{\partial \tau} = \frac{1}{(c-x)^2} \cdot \frac{\partial x}{\partial \tau} < 0
\]

As for the welfare effect, we have:

\[
\frac{\partial W}{\partial \tau} = (b+3).2q_1 \frac{\partial q_1}{\partial \tau} + 4q_2 \frac{\partial q_2}{\partial \tau} - 2r'(x) \frac{\partial x}{\partial \tau}
\]

Substituting (32) and the results derived above into this equation and simplifying we get:

\[
\frac{\partial W}{\partial \tau} = \frac{4[\tau-s(b+2)^2]}{(b+2)^2(1-b)}
\]

Note that the denominator of this fraction is positive. Given the range of value of \( s \) in (31) we can work out that:

\[-(1-b)(\alpha - \tau) \leq \tau - s(b+2)^2 \leq -(1-b)(\alpha - \tau - c) \]
This means that \( \tau - s(b + 2)^2 < 0. \) Hence, we can conclude \( \frac{\partial W}{\partial \tau} < 0. \)

The results that trade liberalization induces higher R&D spendings of firms and, hence, lead to the improvement of their productivity as well as the industry productivity are in sharp contrast with the case of no domestic sales investigated under Proposition 4. This is because both effects of trade liberalization, direct and indirect, complement each other in promoting R&D investments.

Regarding the sale and welfare impacts, the results are qualitatively similar to those under Proposition 4 (and the same as those under Proposition 7). Trade liberalization in the export market is not only welcome by exporting firms as they can expand their output but also by their host country. This is because it makes the domestic economy as a whole become more efficient and reap more welfare.

4 Conclusion

In this paper we consider different scenarios of exporting firm competition to explore the effect of trade liberalization in the foreign market and R&D policy on firms’ incentive to innovate and social welfare. In particular, we study in details the international setting in which firms invest in R&D and sell their differentiated products in a foreign market. The government uses R&D subsidy as a policy tool to maximize social welfare. We show that when firms are independent monopolies, there is no need for the government to subsidize R&D. However, when firms produce substitutable products, it is optimal for the government to tax R&D instead of subsidizing it. Trade liberalization induces the government to tax R&D more heavily as this policy response improves the domestic welfare. Similarly, an increase in the degree of substitutability of goods induced higher optimal R&D taxation.

In the next step, we examine if there are any changes in results when firms also sell their products in the home market. It is found that the optimal policy for the government in this case is always to provide financial support to firms’ R&D activity (positive R&D subsidy) even when firms are independent monopolies. Trade liberalization triggers a higher level of this subsidy when goods are completely different. However, when goods are imperfect substitutes, whether trade liberalization decreases or increases the subsidy depends on the convexity of the R&D cost function. An increase
in the degree of substitutability of makes it optimal for the government to reduce R&D subsidy to firms.

Although the settings explored change from independent monopolies to duopolistic competition and from foreign market to both home and foreign markets, all in all, we find that trade liberalization is always welfare enhancing as it induces higher output sales, both at home and overseas, of firms. It also entails a higher level of cost-reducing R&D spending which then leads to an improvement of firms’ and industry productivity. The only exceptional case where trade liberalization has no impact on R&D investment and productivity is when there is rivalry between exporting firms in the foreign market. In this case, the direct and indirect effects of trade liberalization cancel out each other resulting in no change in R&D investment.

Overall, the results of our model are broadly in line with the literature stressing the complementarity between innovation and export: firms are more likely to export if they innovate and are more likely to innovate if they find good export opportunities (e.g. Lileeva and Trefler, 2010; Bustos, 2010). Although the attention in this paper is restricted to the competition of only two firms, the model can easily be extended to a multiple firm setting. With regards to future research, R&D spillovers between heterogenous firms may be considered to enrich the model.

References


