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Modeling the dynamics of European carbon futures price: a Zipf analysis

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Abstract: This article investigates the European carbon futures price dynamics by applying the Zipf analysis. The results show that:

first, carbon price behaviour is asymmetric, and the long-term bearish probability is greater than the long-term bullish probability.

Second, time-scales of investment and speculators’ expectations of returns have dual effects on carbon price behaviour. The longer time-scales of investment, the higher the bearish probability. The lower expectations of returns, the smaller the distortion of carbon price behaviour.

Third, the differences in carbon market cognitions from non-greedy speculators with different expectations of returns mainly lie in the amplitudes and occasions of carbon price fluctuations, rather than carbon price fluctuations themselves. Fourth, speculators’ expectations of returns have critical points. Once the critical points are reached, they will no longer be able to distort carbon price behaviour.

Finally, we discuss some investment advice for supports of the decision-makers. For non-greedy-type speculators, they will choose to hold negatively in the short term and buy and hold in the long term, while for greedy-type speculators they will sell their European Union Allowances (EUA)s in the short term, and buy and hold in the long term. The results are helpful to hedge against unwanted carbon price movements, and to understand the transactions between different types of agents.

Keywords: EU ETS; carbon futures price; Zipf analysis; expectation of return; time-scale of investment

1. Introduction

Global climate change has become one of the biggest challenges for sustainable development. To implement the Kyoto protocol which provided greenhouse gas emissions reduction targets at the lowest cost, the European Union (EU) has created the EU Emissions Trading System (EU ETS). The EU ETS was founded in January 2005, setting up a Carbon dioxide (CO\textsubscript{2}) emissions cap for the EU’s 12,000 facilities. At present, in terms of market value and transaction volume, the EU ETS carbon futures market is the world’s largest, having become a guide to the global carbon market [1]. Therefore, this work focuses on the EU ETS carbon futures price behaviour analysis.

The carbon market is not only an important tool for human beings to address climate change, but also an important choice for investors to spread their investment risks [2]. Carbon pricing has become one of the key issues of global carbon market developments [3]. As a new market, carbon market is not only governed by market mechanisms, but also affected by unstable environmental factors such as: global climate negotiations, extreme temperatures, financial crises, and other special events, which can induce volatility [4]. In recent years, carbon price behaviour has attracted attention from scholars. Mansanet-Bataller \textit{et al.} [5], Alberola \textit{et al.}[6] and Creti \textit{et al.}[7] used multiple linear regression models with dummy variables to investigate changes in carbon futures price behaviour. Paolella and Taschini [8] and Chevallier
[9] used the GARCH model to examine the volatility of carbon futures price from 2005 to 2007. Benz and Truck[10] used the MS-AR-GARCH model to explore the volatility of carbon futures price returns between 2005 and 2007. Chevallier [11], Chevallier and Sevi [12], and Conrad et al. [13] respectively used the FAVAR, HAR-RV, and FIAPGARCH models to model the volatility of the carbon futures price. Feng et al. [14] used a non-linear method to analyse the volatility of carbon futures price returns. Zhu [15] used empirical mode decomposition (EMD) combined with an artificial neural network (ANN) modelling to predict the EU ETS carbon futures price. Zhu and Wei [16] introduced least squares support vector machines (LSSVM) into their EU ETS carbon futures price forecasts, which performed over ARIMA and ANN models. These studies provide us with important references to understand carbon futures price fluctuations, however, they rarely explore the information of carbon futures price fluctuations (up or down) at different time-scales of investments and different speculator’s expectations of returns. Thus, they cannot provide abundant information to support decision-making.

In this article, we apply the Zipf analysis for the EU ETS carbon futures price to explore the dynamic behaviour of carbon futures price at different time-scales of investments, and following different speculator’s expectations of returns. By doing so, we obtain abundant information about carbon futures price fluctuations. The Zipf analysis, initially used in the study of natural language [17, 18], has been applied to examine the behaviour of financial markets in recent years. More details can be found in Refs. [19-24]. It has also been applied recently in the field of applied statistics by Niu and Wang [25]. However, it has not been yet applied to the analysis of carbon markets to date. We fill this gap in the literature.

The Zipf analysis maps the carbon price fluctuations, upward and downward, using 1 and -1 respectively to generate a new string sequence containing fundamental information of price fluctuations to investigate the dynamic behaviour of the original price sequence. When the carbon price dynamic behaviour follows a random walk, a probability of 1 should be the same as that of -1. When the probability of 1 is not same as that of -1, this indicates that carbon price dynamic behaviour has the characteristics of a trend and long-term memory. It appears beneficial when trying to understand the price formation mechanism of carbon futures, so as to better avoid carbon price risk, and to better inform future transactions.

The remainder of the article is structured as follows. Section 2 presents the Zipf methodology. Section 3 contains the empirical analyses. Section 4 discusses the results obtained. Section 5 briefly concludes.

2. Methodology

In this section, we detail first the Zipf methodology [17, 18], and second, the main parameters of interest.

2.1. Zipf analysis

Zipf analysis, originally introduced in the context of natural languages, has been applied to various types of data in
physical and social sciences. To study the fluctuation of carbon price changes, in this section, we apply the Zipf analysis to investigate the symbolic dynamics of real data from the EU ETS carbon futures market.

\[ p(t) = \{p(t_1), p(t_2), \ldots, p(t_n)\} \]

is the original carbon price time series, and \( r_i(\tau) \) is the return of day \( i \) under time-scale of investment \( \tau \). The definition of \( \tau \)-returns of carbon price is given by:

\[
r_i(\tau) = \left( \frac{p(t_i + \tau) - p(t_i)}{p(t_i)} \right), \quad i = 1, 2, \ldots, n - \tau
\]

From which we get the \( \tau \)-return series \( r(\tau) = \{r_1(\tau), r_2(\tau), \ldots, r_{n-\tau}(\tau)\} \). Next, we map the \( \tau \)-return series into a new three-alphabet symbolic sequence \( f_i(\tau, \epsilon) \):

\[
f_i(\tau, \epsilon) = \begin{cases} 
-1, & \text{if } r_i \leq -\epsilon \\
0, & \text{if } -\epsilon < r_i < +\epsilon \\
1, & \text{if } r_i \geq +\epsilon 
\end{cases}
\]

We obtain a new sequence which contains the most basic information of price-up, price-stable and price-down \( f(\tau, \epsilon) = \{f_1(\tau, \epsilon), f_2(\tau, \epsilon), \ldots, f_{n-\tau}(\tau, \epsilon)\} \), with \( \tau \) a given time-scale of investment. Especially, when \( \tau = 1, 5, 20, 60, 120, \text{ or } 250 \), these are called characteristic time-scales, which approximately stand for one transaction day (TD), one transaction week, one transaction quarter, one transaction half year and one transaction year, respectively, in terms of business time (units with weekends and holidays eliminated). 1, 0 and -1 denote ‘price-up’, ‘price-stable’ and ‘price-down’, respectively. \( \epsilon \), a psychological threshold for investors, can be interpreted as the expected returns for investors. Especially, when \( \epsilon = 0, 0.05, \text{ or } 0.1 \), speculator’s expectations of returns are 0, 0.05 or 0.1, respectively.

To further explore the information of carbon futures price behaviour, we introduce absolute frequencies and relative frequencies to examine carbon price behaviours of \( f(\tau, \epsilon) \) for different values of parameters \( \epsilon \) and \( \tau \). We define the frequencies of occurrences for price-up, price-stable and price-down as \( n_+(\tau, \epsilon), n_0(\tau, \epsilon) \) and \( n_-(\tau, \epsilon) \) respectively, so \( n_+(\tau, \epsilon) + n_0(\tau, \epsilon) + n_-(\tau, \epsilon) = n - \tau \). We define the absolute frequencies of \( f_i(\tau, \epsilon) \) as

\[
p_-(\tau, \epsilon) = \frac{n_-(\tau, \epsilon)}{n - \tau} \\
p_0(\tau, \epsilon) = \frac{n_0(\tau, \epsilon)}{n - \tau} \\
p_+(\tau, \epsilon) = \frac{n_+(\tau, \epsilon)}{n - \tau}
\]

and the corresponding relative frequencies as

\[
\Phi_-(\tau, \epsilon) = \frac{n_-(\tau, \epsilon)}{n_+(\tau, \epsilon)} \\
\Phi_+(\tau, \epsilon) = \frac{n_+}\n_2(\tau, \epsilon)
\]

(4)
where \( n_+(\tau, \varepsilon) = n_-(\tau, \varepsilon) + n_+(\tau, \varepsilon) \). In the definition of the relative frequencies, we neglect occurrences of price-stable, and use \( \Phi_+(\tau, \varepsilon) \) and \( \Phi_-(\tau, \varepsilon) \) to measure the occurrences of price-up and price-down respectively. In the following discussion, for the actual carbon price data, we consider the statistical properties of absolute frequencies and relative frequencies for different values of the two parameters \( \varepsilon \) and \( \tau \).

2.2. Economic significance of the parameters \( \varepsilon \) and \( \tau \)

In the EU ETS market, there are different types of speculators, such as suppliers, energy consumers, governments, banks, and so on. Among them, their respective influences, derived from speculators’ behaviour and psychology, are difficult to quantitatively analyse and forecast. Different time-scales of investment lead to different expectations of returns. The parameter \( \tau \) is designed to reflect speculator’s average transaction time interval. The higher \( \tau \), the longer speculator’s transaction time interval. The parameter \( \varepsilon (\varepsilon \geq 0) \) is designed as speculators’ psychological threshold of expected returns. The maximum earnings expectation is \( +\varepsilon \), and the maximum risk value is \( -\varepsilon \). To simplify, we assume that the absolute values of \( \pm\varepsilon \) are equal. Usually, speculators believe that the current carbon price fluctuations depend on their psychological threshold \( \varepsilon \). Due to the presence of market transaction costs and uncertainties, speculators believe that when returns are higher than \( +\varepsilon \), carbon price shows a substantial rise. After reaching their prospective earnings, they will sell their own European Union Allowances (EUAs). When the return is between \( \pm\varepsilon \), speculators believe that carbon price has not undergone a substantial change, and they will not participate in market trading and choose to continue to hold negatively. When the return is inferior to \( -\varepsilon \), speculators believe that carbon price is about to undergo a substantial decrease, and they will opt for the buy-and-hold strategy. Thus, in fact the parameter \( \varepsilon \) can reflect speculators’ psychological endurance concerning carbon price fluctuations, as well as their optimism about future market trends.

From the perspective of speculators, short-term speculators need to watch for each peak and trough of carbon futures price fluctuations, which constitutes their profit pattern. Their frequency of attention to carbon market is much higher than long-term investors. Long-term investors are concerns about the long-term trend of carbon price fluctuations, thus their frequencies of attention to carbon market are lower, and their values of \( \tau \) are therefore higher. According to whether carbon price achieves speculators’ desired returns, they correspondingly make their decisions. Usually, when carbon price does not match speculators’ expected returns, they cannot produce trading desire. Of course, not all speculators’ decisions are rational, but when making so-called irrational decisions, speculators are changing their expected returns in fact.

3. Empirical analyses

3.1. Data
In this article, we use the EUA carbon futures price data from the European Climate Exchange (ECX), the most liquid carbon futures market under the EU ETS, from 22 April, 2005 to 26 October, 2012, totalling of 1,929 daily observations, as shown in Fig.1. According to the Eqs (1) to (4), we pre-process the raw data, and obtain the returns sequence of carbon price itself, price-up and price-down under different $\varepsilon$ and $\tau$, as well as the absolute and relative frequencies of each sequence, respectively.

![The ECX carbon futures price series](image)

3.2. The influences of $\varepsilon$ and $\tau$

According to the Eq (2), we map the return sequences into string sequences. We conduct the deviation (namely 1 frequency minus -1 frequency) analysis under different $\varepsilon$ and $\tau$: the results are shown in Fig. 2.

When only considering $\tau$, without considering $\varepsilon$, i.e., $\varepsilon=0$, the results unfold as follows. First, because the deviation in each $\tau$ is non-zero, $\tau$ has an important effect on carbon price fluctuations. Second, although in the extreme case: $\varepsilon=0$ and $\tau=1$, we observe a deviation of 0.00934 between the probabilities of price-up and price-down. This implies that the carbon price fluctuations are asymmetric.

Third, there is a negative deviation within each characteristic $\tau$, and this deviation increases with further oscillations.

Fourth, speculators’ $\varepsilon$ for price-up is greater than for price-down. With the extension of $\tau$, speculators generally believe that carbon price would decrease in the long-term.

When considering $\varepsilon$, i.e., $\varepsilon>0$, we can draw the following conclusions. First, speculators’ $\varepsilon$ has a dramatic influence on carbon price fluctuations.

Second, different $\varepsilon$ such as $\varepsilon=0.05$ and $\varepsilon=0.1$, can cause deviations under different $\tau$.

Third, with increased $\tau$, deviations decrease below those corresponding to the absence of consideration of $\varepsilon$. This result implies that $\varepsilon$ can distort carbon price fluctuations for a certain period, further promoting price
This preliminary analysis reveals that $\varepsilon$ and $\tau$ can influence carbon price behaviour. How can we determine $\tau$ and measure the effect of different $\varepsilon$ on carbon price behaviour? How can we measure the effect of speculators’ $\varepsilon$ on carbon price behaviour? To answer these questions, further work is needed.

Fig. 2. The deviations of probabilities of price-up and price-down under characteristic time-scales

3.3 Division of speculators based on $\varepsilon$

With the increase of $\varepsilon$, how will the absolute and relative frequencies change? We compute the absolute and relative frequencies under different $\varepsilon$ ($0 < \varepsilon < 1$) by increments of 0.05: the corresponding results are shown in Table 1, and Figs 3 and 4.
Fig. 3. The evolution of absolute frequencies under different $\varepsilon$. 

- **a.** Absolute frequency $p_0(\tau, \varepsilon)$ for different values of $\varepsilon$.
- **b.** Absolute frequency $p_0(\tau, \varepsilon)$ for different values of $\varepsilon$.
- **c.** Absolute frequency $p^+(\tau, \varepsilon)$ for different values of $\varepsilon$.
- **d.** Deviation $\tau$ for different values of $\varepsilon$. 

The graphs illustrate how the absolute frequencies change with respect to different values of $\varepsilon$ and $\tau$. The evolution of these frequencies under varying conditions is crucial for understanding the behavior of the system in question.
Fig. 2 shows that speculators with different $\varepsilon$ may have different perceptions of carbon price fluctuations, so it is necessary to classify speculators and explore the characteristics of different categories thereof. Figs 3 and 4 show that, with the evolution of $\varepsilon$, the absolute and relative frequencies tended to be saturated respectively. The critical points, namely $\varepsilon_c(\tau)$, are presented in Table 1. Each frequency has reached saturation after its critical points.

**Table 1. Various frequencies’ critical points**

<table>
<thead>
<tr>
<th>$\varepsilon_c(\tau)$</th>
<th>$p_-(\tau,\varepsilon)$</th>
<th>$p_0(\tau,\varepsilon)$</th>
<th>$p_+(\tau,\varepsilon)$</th>
<th>$\Phi_-\left(\tau, \varepsilon\right)$</th>
<th>$\Phi_+\left(\tau, \varepsilon\right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau=1$</td>
<td>0.3</td>
<td>0.3</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>$\tau=5$</td>
<td>0.4</td>
<td>0.4</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>$\tau=20$</td>
<td>0.4</td>
<td>0.6</td>
<td>0.6</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>$\tau=60$</td>
<td>0.6</td>
<td>1</td>
<td>1</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>$\tau=120$</td>
<td>0.7</td>
<td>0.8</td>
<td>0.8</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>$\tau=250$</td>
<td>0.7</td>
<td>0.7</td>
<td>0.6</td>
<td>0.8</td>
<td>0.7</td>
</tr>
</tbody>
</table>
Although speculators’ $\varepsilon$ can distort carbon price fluctuations, this distortion is not indefinite. As with the growth of $\varepsilon$, there is a saturation point. After saturation, frequencies of increase and decrease remain stable. This means that once speculators’ $\varepsilon$ reaches a certain level, it will no longer be able to distort carbon price behaviour.

Under different $\varepsilon$, we obtain the following results. First, the higher speculators’ $\varepsilon$, the more extreme the effect on judgment of carbon price behaviour. For instance, when $\tau = 20$ and $\varepsilon < 0.2$, carbon price behaviour is almost random. When $\varepsilon \geq 0.2$, the probability of price-up or price-down is almost equal to zero.

Second, the deviations between price-up and price-down under different $\varepsilon$ are different at the same $\tau$. Under the same $\tau$, with the improvement of $\varepsilon$, the deviations become larger, until reaching saturation.

Third, there is an asymmetry to carbon price fluctuations. Before reaching the saturation point, the deviation between price-up and price-down is non-zero, but the probability of decreasing is slightly higher than that of increasing. For example, with a higher $\varepsilon$ (0.2) and a longer $\tau$ (250 TD), the probability of decreasing is higher than that of increasing by 21.08%.

According to Fig. 4, speculators can be roughly divided into two categories: greedy ($\varepsilon \geq 0.2$) and non-greedy ($\varepsilon < 0.2$) speculators. For speculators who have smaller $\tau$ (1, 5, or 20), when $\varepsilon < 0.2$ (non-greedy), the relative frequencies are approximately 0.5, and the probability of price-up is similar to that of price-down, i.e., carbon price behaviour is close to a random walk. However, when $\varepsilon \geq 0.2$ (greedy), the relative frequencies quickly converge to the extreme value of 0 or 1. For speculators who have longer $\tau$ (60, 120, or 250), the relative frequencies also show changes from slow to high growth before and after $\varepsilon = 0.2$.

3.4 Absolute frequencies of carbon price fluctuations

According to the Eq (3), we obtain the absolute frequencies, namely $p_s(\tau, \varepsilon)$, $p_0(\tau, \varepsilon)$ and $p_r(\tau, \varepsilon)$. Fig. 3 shows the evolution of absolute frequencies under different $\tau$, when $\varepsilon = 0.05, 0.1, 0.2, 0.3, 0.4$, and 0.5. To quantify speculators’ cognitions of carbon price behaviour, and distortions from current carbon price behaviour, we take $\varepsilon = 0$ as a reference, namely taking $p_s(\tau, 0)$, $p_0(\tau, 0)$ and $p_r(\tau, 0)$ as the references, and $p_s(\tau, \varepsilon)$ as the distortion for speculators’ cognitions of historical carbon price information. The corresponding results are shown in Table 2. Fig. 5 shows that there are some distortions in speculators’ cognitions of carbon price behaviour.
Fig. 5. Speculators’ cognitions of historical carbon price information (absolute frequencies)

Greedy and non-greedy speculators have similar cognitions of historical carbon price information. In contrast, Fig. 5 shows that non-greedy speculators have the complete opposite cognitions of price-down and price-stable. Fig. 5a shows that non-greedy speculators’ cognitions of price-down are rising. Fig. 5b shows that non-greedy speculators’ cognitions of price-stable are falling. In the meantime, Fig. 5c shows that speculators’ cognitions of price-down frequencies exhibit inflection points. The corresponding results are shown in Table 3.

Table 4 shows that the distortions, caused by speculators’ $\epsilon$ to $p_0(\tau,0)$, are generally positive. In contrast, Tables 5 and 6 show that the distortions, caused by speculators’ $\epsilon$ to $p_-(\tau,0)$ and $p_+(\tau,0)$, are both negative. Fig. 5 and Tables 4 to 6 show that with increased $\tau$, the distortions caused by speculators’ $\epsilon$ generally tends to decrease.
### Table 2. Actual historical carbon price information under characteristic time scales

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Time-scales (TD)</th>
<th>$\tau = 1$</th>
<th>$\tau = 5$</th>
<th>$\tau = 20$</th>
<th>$\tau = 60$</th>
<th>$\tau = 120$</th>
<th>$\tau = 250$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_-(\tau,0)$</td>
<td>0.50467</td>
<td>0.48857</td>
<td>0.51231</td>
<td>0.56233</td>
<td>0.51354</td>
<td>0.59738</td>
<td></td>
</tr>
<tr>
<td>$p_0(\tau,0)$</td>
<td>0.93517</td>
<td>0.65852</td>
<td>0.35778</td>
<td>0.23756</td>
<td>0.14649</td>
<td>0.14354</td>
<td></td>
</tr>
<tr>
<td>$p_+(\tau,0)$</td>
<td>0.49533</td>
<td>0.51144</td>
<td>0.48769</td>
<td>0.43767</td>
<td>0.48646</td>
<td>0.40262</td>
<td></td>
</tr>
<tr>
<td>$\Phi_-(\tau,0)$</td>
<td>0.50467</td>
<td>0.48857</td>
<td>0.51231</td>
<td>0.56233</td>
<td>0.51354</td>
<td>0.59738</td>
<td></td>
</tr>
<tr>
<td>$\Phi_+(\tau,0)$</td>
<td>0.49533</td>
<td>0.51144</td>
<td>0.48769</td>
<td>0.43767</td>
<td>0.48646</td>
<td>0.40262</td>
<td></td>
</tr>
</tbody>
</table>

### Table 3. The critical points of $p_+(\tau,\varepsilon)$ under non-greedy expectations

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>$\varepsilon = 0$</th>
<th>$\varepsilon = 0.05$</th>
<th>$\varepsilon = 0.1$</th>
<th>$\varepsilon = 0.15$</th>
<th>$\varepsilon = 0.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_c(\tau)$</td>
<td>120</td>
<td>120</td>
<td>120</td>
<td>120</td>
<td>250</td>
</tr>
</tbody>
</table>

### Table 4. The distortions of speculators’ $\varepsilon$ to $p_0(\tau,0)$ under characteristic time scales

<table>
<thead>
<tr>
<th>Absolute</th>
<th>Time-scales of investment (TD)</th>
<th>$\tau = 1$</th>
<th>$\tau = 5$</th>
<th>$\tau = 20$</th>
<th>$\tau = 60$</th>
<th>$\tau = 120$</th>
<th>$\tau = 250$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon = 0.05$</td>
<td>0.43983</td>
<td>0.14709</td>
<td>-0.12991</td>
<td>-0.20011</td>
<td>-0.33997</td>
<td>-0.25908</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon = 0.1$</td>
<td>0.49689</td>
<td>0.38462</td>
<td>0.14248</td>
<td>-0.01498</td>
<td>-0.15644</td>
<td>-0.13163</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon = 0.2$</td>
<td>0.50156</td>
<td>0.47765</td>
<td>0.40440</td>
<td>0.26859</td>
<td>0.14152</td>
<td>0.03157</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon = 0.3$</td>
<td>0.50467</td>
<td>0.48493</td>
<td>0.48612</td>
<td>0.42322</td>
<td>0.30404</td>
<td>0.17808</td>
<td></td>
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<tr>
<td>$\varepsilon = 0.4$</td>
<td>0.50467</td>
<td>0.48857</td>
<td>0.50498</td>
<td>0.51043</td>
<td>0.40575</td>
<td>0.33830</td>
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</tr>
<tr>
<td>$\varepsilon = 0.5$</td>
<td>0.50467</td>
<td>0.48857</td>
<td>0.50812</td>
<td>0.51231</td>
<td>0.48646</td>
<td>0.40262</td>
<td></td>
</tr>
</tbody>
</table>

### Table 5. The distortions of speculators’ $\varepsilon$ to $p_-(\tau,0)$ under characteristic time scales

<table>
<thead>
<tr>
<th>Absolute</th>
<th>Time-scales of investment (TD)</th>
<th>$\tau = 1$</th>
<th>$\tau = 5$</th>
<th>$\tau = 20$</th>
<th>$\tau = 60$</th>
<th>$\tau = 120$</th>
<th>$\tau = 250$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon = 0.05$</td>
<td>-0.47147</td>
<td>-0.31653</td>
<td>-0.18596</td>
<td>-0.13965</td>
<td>-0.07376</td>
<td>-0.06226</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon = 0.1$</td>
<td>-0.50104</td>
<td>-0.43035</td>
<td>-0.31598</td>
<td>-0.24773</td>
<td>-0.15644</td>
<td>-0.10899</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon = 0.2$</td>
<td>-0.50311</td>
<td>-0.48181</td>
<td>-0.45940</td>
<td>-0.40075</td>
<td>-0.27695</td>
<td>-0.20905</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon = 0.3$</td>
<td>-0.50467</td>
<td>-0.48545</td>
<td>-0.5034</td>
<td>-0.48529</td>
<td>-0.35434</td>
<td>-0.28529</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon = 0.4$</td>
<td>-0.50467</td>
<td>-0.48857</td>
<td>-0.51231</td>
<td>-0.54254</td>
<td>-0.42454</td>
<td>-0.37999</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon = 0.5$</td>
<td>-0.50467</td>
<td>-0.48857</td>
<td>-0.51231</td>
<td>-0.56126</td>
<td>-0.48480</td>
<td>-0.51757</td>
<td></td>
</tr>
</tbody>
</table>
Table 6. The distortions of speculators’ $\varepsilon$ to $p_+(\tau,0)$ under characteristic time scales

<table>
<thead>
<tr>
<th>Absolute frequency</th>
<th>Time-scales of investment (TD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon = 0.05$</td>
<td>-0.46369 -0.34200 -0.17182 -0.09791 -0.07242 -0.08100</td>
</tr>
<tr>
<td>$\varepsilon = 0.1$</td>
<td>-0.49118 -0.4657 -0.31378 -0.17496 -0.17302 -0.16200</td>
</tr>
<tr>
<td>$\varepsilon = 0.2$</td>
<td>-0.49326 -0.50728 -0.43269 -0.30551 -0.35047 -0.22513</td>
</tr>
<tr>
<td>$\varepsilon = 0.3$</td>
<td>-0.49533 -0.51091 -0.47040 -0.37560 -0.43615 -0.29541</td>
</tr>
<tr>
<td>$\varepsilon = 0.4$</td>
<td>-0.49533 -0.51143 -0.48036 -0.40556 -0.46766 -0.36033</td>
</tr>
<tr>
<td>$\varepsilon = 0.5$</td>
<td>-0.49533 -0.51143 -0.48350 -0.42429 -0.47706 -0.39011</td>
</tr>
</tbody>
</table>

3.5 Relative frequencies of carbon price fluctuations

According to the Eq (4), we obtain the relative frequencies, as shown in Figs 6 and 7. Due to $\Phi_+ (\tau, \varepsilon) + \Phi_-(\tau, \varepsilon) = 1$, we only discuss $\Phi_+ (\tau, \varepsilon)$. Similarly to Section 3.4, we investigate the distortion of speculators’ $\varepsilon$ to historical carbon price information in terms of relative frequencies. The results are reported in Table 7. In contrast with absolute frequencies, the distortions caused by speculators’ $\varepsilon$ are mostly negative. Only a fraction of them are positive, and negative distortions are more likely to occur over longer $\tau$ ($\tau \geq 120$) than smaller $\tau$ ($\tau \leq 5$).

Table 7. The distortions of speculators’ $\varepsilon$ to $\Phi_+(\tau,0)$ under characteristic time-scales

<table>
<thead>
<tr>
<th>$\Phi_+ (\tau, \varepsilon)$</th>
<th>Time-scales of investment (TD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon = 0.05$</td>
<td>-0.00733 -0.01524 0.004154 0.007947 -0.00167 -0.02736</td>
</tr>
<tr>
<td>$\varepsilon = 0.1$</td>
<td>0.038001 -0.07143 -0.0181 0.017384 -0.01863 -0.07256</td>
</tr>
<tr>
<td>$\varepsilon = 0.2$</td>
<td>0.076097 -0.13048 0.022019 0.012242 -0.12147 -0.08894</td>
</tr>
<tr>
<td>$\varepsilon = 0.3$</td>
<td>-0.49532 -0.36858 0.17231 0.008487 -0.24635 -0.14694</td>
</tr>
<tr>
<td>$\varepsilon = 0.4$</td>
<td>-0.49532 -0.51144 0.51231 0.180889 -0.3121 -0.23978</td>
</tr>
<tr>
<td>$\varepsilon = 0.5$</td>
<td>-0.49532 -0.51144 0.51231 0.488259 -0.24008 -0.26714</td>
</tr>
</tbody>
</table>
Figs 6 and 7 show that non-greedy and greedy speculators have entirely different cognitions of the historical carbon price information in terms of relative frequencies. The former cognition patterns are generally similar to actual historical information. The latter cognitions patterns differ completely from actual historical information.

Under lower $\varepsilon$, carbon price fluctuations are relatively well understood. When $\varepsilon < 0.2$, the probabilities of price-up and price-down are accurate, following two stages. During the first stage, carbon price fluctuates more slowly, and relative probabilities are found between 0.4 and 0.6. Thus, carbon price fluctuations correspond to carbon market’s spontaneous behaviour as a result of market mechanism. During the second stage, we observe impacts from heterogeneous events. The global financial crisis in 2008 and European debt crisis in 2011 led to a reduction in manufacturing output, a corresponding reduction in EUA demand, and carbon price fell sharply. Although the influences of these heterogeneous events are in long-term, their probabilities of occurrence are small.
Under higher $\varepsilon$, carbon price fluctuations are hard to be understood. When $\varepsilon \geq 0.2$, the relative probabilities become relatively scattered, and quickly converge to 0 or 1. The probabilities of price-up and price-down are so unstable that they lead to large deviations. Investment returns become unstable. As a consequence, speculators’ $\varepsilon$ cannot be too high.

4. Results analysis and discussion

Due to speculators’ decisions being based on their cognitions of historical information, we discuss the influences of $\tau$ and $\varepsilon$ on carbon price formation mechanism following our Zipf analysis.

First, at a specific $\tau$, the absolute and relative frequencies converge to their saturation points. Although speculators can distort carbon price behaviour, this distortion will not induce unlimited growth. With the growth of $\varepsilon$, there is a saturation point existed in the absolute and relative frequencies, respectively. The frequencies are no longer growing beyond, implying that once speculators’ expectations reach a certain higher level, they will no longer be able to distort carbon price behaviour. Owing to carbon price fluctuations, speculators’ unreasonably higher expectations cannot be met. Unless speculators modify their expectations, it is disadvantageous to trade. In addition, short-term ($\tau \leq 20$) speculators’ judgments on carbon price fluctuations tend to extremes. When $\tau = 1$, non-greedy speculators tend to believe that carbon price is close to a random walk, and it becomes difficult to judge any trend therein, whereas greedy speculators argue that the possibility of price-up is 100%.

Second, in terms of the absolute frequencies, $\varepsilon$ has a distorting effect on carbon price behaviour. The higher $\varepsilon$, the larger the distortion of real historical information. Further, the distortions to $p_0(\tau, \varepsilon)$ are mostly positive, while the distortions to other frequencies are negative. With increased $\tau$, the distortions between $p_+ (\tau, 0)$ and $p_0 (\tau, \varepsilon)$ tend to decrease. Non-greedy speculators’ cognitions of historical carbon price information exhibit similarities and internal consistencies with current carbon price trends. Although speculators’ expectations can distort carbon price behaviour, they cannot affect the long-term movement of carbon price. The differences in market cognitions, arising from different expectations of speculators, are mainly amplitudes and occasions of carbon price fluctuations, rather than carbon price fluctuations themselves. Speculators’ cognitions for long-term carbon price-down tend to be more consistent. They believe that the probability of price-stable reduces with increased $\tau$. However, speculators’ cognitions for price-up have obvious inflection points. Speculators’ cognitions have fundamental changes before and after inflection points. Before inflection points, speculators believe that carbon price exhibits a likelihood of rising. The lower $\varepsilon$, the greater the confidence in a short-term price-up. After inflection points, they usually believe that the possibility of price-up is smaller, and they cannot make more profits. For speculators with lower returns, there is a high probability that carbon price will rise in the short-term. However, in the long-term, this probability tends to decline, i.e., with the extension of $\tau$, speculators tend to believe that carbon price is bearish.
Because two types of speculators have different cognitions of carbon market, we discuss non-greedy and greedy speculators separately with regards of their relative frequencies. For non-greedy speculators, \( \Phi_+(\tau, \varepsilon) \) lies mostly under 0.5, and \( \Phi_-(\tau, \varepsilon) \) lies mostly above 0.5. With increased \( \tau \), \( \Phi_+(\tau, \varepsilon) \) decreases, which means that carbon price behaviour is biased and exhibits a negative trend. The long-term bearish probability is bigger, which corroborates the aforementioned deviation analysis of absolute frequencies. Speculators’ expectations have distorting effects on carbon price behaviour. When \( \varepsilon = 0.1 \), the probability of price-down is less than 0.5 within a trade week (\( \tau \leq 5 \)), but the probability of price-up is above 0.5. \( \Phi_+(\tau, \varepsilon) \) and \( \Phi_-(\tau, \varepsilon) \) has obvious inflexion points. Short- and medium-term investments (\( 1 \leq \tau \leq 120 \)) show changing probabilities of carbon price nearing 0.5, and stabilising between 0.4 and 0.6. At this moment, carbon price fluctuations are relatively close to a random walk, and the probability of price-up is the same as that of price-down. Hence, long-buying and short-selling becomes a potentially profitable trading strategy. Permanent investment (\( \tau > 120 \)) exhibits an unstable carbon price behaviour, as the probability of price-down becomes bigger. When the probability of price-up greatly decreases, speculators tend to believe that carbon price is bearish in the long-term. Although there remains the possibility for short-term bullishness, speculators tend to believe that carbon price will be bearish in the long-term. When \( \varepsilon = 0.1 \) and \( \tau \leq 5 \), there are \( \Phi_+(\tau, \varepsilon) \geq 0.5 \) and \( \Phi_-(\tau, \varepsilon) < 0.5 \), which are different from the price perceptions of speculators who believe that \( \varepsilon = 0.15 \) or 0.2.

For greedy speculators, in the short-term, speculators have volatile perceptions of carbon price fluctuations. The greedier the speculator, the greater the volatility. \( \Phi_+(\tau, \varepsilon) \) lies mostly above 0.5, and \( \Phi_-(\tau, \varepsilon) \) lies mostly below 0.5. With increased \( \tau \), \( \Phi_-(\tau, \varepsilon) \) increases, which means that carbon price behaviour is biased and exhibits a negative trend, i.e., the long-term bearish probability is higher, which is also consistent with non-greedy speculators. Greedy speculators and non-greedy speculators have entirely different perception patterns of short-term carbon price fluctuations. Speculators with \( \varepsilon = 0.3 \) believe that there are great deviations between 0 to 1 during \( 1 \leq \tau \leq 5 \), which shows that greedy speculators’ risk propensities are so strong that they are irrational. Speculations based on this acknowledged facet of carbon market are likely to aggravate carbon price volatilities, market risks, and uncertainties. However, aggravated carbon price volatilities are likely to be greeted by higher expected speculators, if they want to achieve their expectations. Overall, the bearish probability of these speculators on carbon price is still higher than their bullish probability.

5. Concluding remarks

Based on the above Zipf analysis, we can draw the following conclusions. First, carbon price behaviour is asymmetric. Whatever \( \varepsilon \) and \( \tau \), the long-term bearish probability is, on the whole, greater than the long-term bullish probability.
Second, $\varepsilon$ and $\tau$ have dual effects on carbon price behaviour. The longer $\tau$, the higher the bearish probability. The lower $\varepsilon$, the smaller the distortions of carbon price behaviour. Speculators with $\tau \leq 20$ tend to two extremes in their acknowledgements of carbon price fluctuations: non-greedy speculators believe that carbon price fluctuation is close to a random walk, whereas greedy speculators believe that carbon price is either 100% bullish or 100% bearish. Speculators with $\tau \geq 60$ are relatively less extreme. They believe that carbon price will remain bearish in the long-term, and with the growth of $\varepsilon$, that the bearish probability slowly increases.

Third, the differences in carbon market cognitions from non-greedy speculators with different expectations mainly lie in the amplitudes and occasions of carbon price fluctuations, rather than carbon price fluctuations themselves. Compared with non-greedy speculators, greedy speculators have entirely different perception patterns of carbon market, being very unstable and tending to go to extremes. When speculating based on their acknowledged perception of carbon market, they will greatly distort carbon price and aggravate its volatilities, therefore increasing market uncertainties.

Fourth, there are critical points existing in the speculators’ expected returns. Once the critical points are reached, they will no longer be able to distort carbon price behaviour, which means that unless speculators modified their higher expectations, they risked being, pardon the colloquialism, wiped out of the carbon market.

Based on the above conclusions obtained, we can provide the decision-makers with some investment advice as follows. First, for non-greedy-type speculators ($\varepsilon < 0.2$), in the short term ($1 \leq \tau < 60$), the probabilities of price-up and price-down are pretty much the same, namely carbon price behavior is close to a random walk. Due to the existence of transaction costs, they will not participate in market trading and choose continue to hold negatively; in the long term ($\tau \geq 60$), the probability of price-down is increasing slowly, and the probability of price-up is declining slowly. Thus, carbon price tends to fall in the long-term, so they will choose to buy and hold. Second, for greedy-type speculators ($\varepsilon \geq 0.2$), and their expected returns lying in the range of critical points, in the short term ($1 \leq \tau < 60$), the probability of price-up quickly converges to 1, and the probability of price-down quickly converges to 0. Thus, carbon price has a substantial rise, they will sell their EUAs; in the long term ($\tau \geq 60$), the probability of price-up quickly converges to 0, and the probability of price-down quickly converges to 1. Thus carbon price has a substantial fall, carbon price tends to decline in the long-term, they will choose to buy and hold.

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