On the detection of extreme movements and persistent behavior in Mediterranean stock markets: a wavelet-based approach

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Abstract

We combine the global Hurst exponent and Morlet wavelet multi-resolution analysis to investigate the dynamic behavior of six selected stock markets in the Mediterranean region. Specifically, we employ the resonance coefficients and their power spectra to identify potential extreme movements and long-term dependence in stock returns. Using weekly data for the period 2005-2010, our results reveal that the wavelet multi-resolution approach is able to reconstruct the effects of major extreme shocks on stock returns of studied markets, such as the Asian financial crisis, the 9/11 terrorist attacks, and the 2007-2009 financial crisis. Moreover, the wavelet-based global Hurst exponent indicates the presence of long-term dependencies in stock returns of all the considered markets, except for France where the anti-persistent behavior is detected. Overall, our findings are useful to assess stock market efficiency and provide new insights into stock market dynamics over different time scales.

Keywords: persistence, Mediterranean markets, wavelet analysis, Hurst exponent, return behavior

JEL classifications: G15, C14, C58
1. Introduction

Stock market dynamics is a central issue to optimal portfolio allocation, hedging strategy and policy regulatory actions. This issue has also received much attention from academics, given the observed complex behavior of stock markets over time. A number of time series models, such as ARIMA and GARCH-type processes, have been developed to capture the dynamic patterns of asset prices. These models are, however, often grounded on the stationarity and the normality of the error terms or return innovations, while non-Gaussian distributions have been well documented for macroeconomic and financial time series (Loretan and Phillips, 1994; van Dijk, 2004). Also, it is now common that standard stationary models are not suitable for detecting non-stationary structural change and long-range dependence. Using traditional models may thus induce misspecifications and misleading conclusions on the true behavior of financial markets.

Long-range dependence in stock markets has been long investigated in the quantitative finance literature and is now recognized as one of the major stylized facts of stock returns. In general, previous works fail to detect evidence of long-range dependence in stock returns for developed markets (e.g., Lo, 1991; Cheung and Lai, 1993; Barkoulas and Baum, 1998). The available evidence on both emerging and transitional markets is rather supportive of the existence of long-range dependence in stock returns (Sadique and Silvapulle, 2001; Wright, 2001).

It is worth noting that spectral density and wavelet analysis appear as promising approaches for the identification of long-range dependence. For example, Beran (1994) adopts a conventional covariance stationary approach with constant integer correlation to separate the “noisy” part from the true patterns in stock return data. Ramsey et al. (1995) suggest a more accurate approach based on the wavelet multi-resolution analysis (MRA) which is particularly suitable for analyzing non-stationary time series with time-varying fractional nonlinear dependencies. These authors note the relevance of wavelets in detecting highly localized structures in the time and frequency domains of stock return behavior.

In this article, we examine whether the dynamics of stock returns in selected markets of the Mediterranean region is governed by a persistent behavior over different time horizons. Our motivation stems from the fact that little has been done on these markets, while financial assets issued by these markets are integrated parts of dedicated diversification strategies of regional investors. For instance, only two studies have focused on the persistent behavior of Latin American and Chinese stocks markets (Kyaw et al., 2006; Los and Yu, 2008). Our analysis thus allows for a comparison of dynamic behavior between emerging and mature
stock markets (Egypt, Turkey, Tunisia, France, Greece and Spain), over a relatively longer period than previous studies, which accounts for the potential impacts of most extreme events since 1995 (e.g., Asian financial crisis 1997-1998, terrorist attacks on September 11, 2001, subprime and global financial crisis 2007-2009, and crude oil price surges in 2008). Furthermore, unlike some previous works including Kyaw et al. (2006), Los and Yu (2008), and Maharaj et al. (2011), we implement the Morlet’s continuous wavelet analysis together with the global Hurst’s exponent to provide comprehensive insights about some critical issues related to Mediterranean stock market dynamics. These issues include, in addition to the long-run dependency characteristics, the detection of extreme return and volatility movements, the validity of efficient capital market hypothesis, and the differences in the dynamic return behavior between selected emerging and developed markets of the Mediterranean region. The results of our investigation are particularly useful for portfolio managers and market authorities as far as they are respectively concerned by accurate portfolio allocation decisions and the improvement of stock market efficiency.

The remainder of the article is organized as follows. Section 2 briefly provides the theoretical background of the long-range dependence and the global Hurst’s exponent. Section 3 reviews the empirical literature devoted to long-term dependencies using MRA approach. Section 4 presents the methodology used to empirically assess the long-range dependence. Section 5 reports and discusses the empirical findings and their implications. Section 6 concludes the article.

2. Long-range dependence and Hurst’s exponent

It is now common that long memory is related to a high degree of persistence of the observed time series. Initially, a number of researchers including Hurst (1951), Mandelbrot and Wallis (1968), Mandelbrot (1972), and McLeod and Hipel (1978) have investigated the long memory dynamics in various domains such as hydrology, geophysics, climatology, and biology. The phenomenon of slowly declining autocorrelation of differenced time series has then gained the attention from researchers in other areas of research and there is now an extensive empirical literature on long memory.¹

The strand of the empirical finance literature concerned with the dynamic behavior of financial markets has also examined their long-range dependence phenomenon. The main ob-

¹ Banerjee and Urga (2005) provided an excellent literature overview in relation to modeling structural breaks, long memory and stock market volatility.
jective is to draw conclusions on market informational efficiency, asset return predictability, volatility spillovers and portfolio’s hedging strategies. From an empirical perspective, the development of the class of fractionally integrated processes was supported by the fact that a number of macroeconomic and financial time series are neither $I(0)$ nor $I(1)$, which suggest the presence of significant autocorrelation up to very long lags, also known as “hyperbolic decay” (Banerjee and Urga, 2010). This is the main characteristic of long memory processes and therefore the integration order must be a fractional number, conventionally noted $d$. Contrary to the $I(0)$ processes and to the $I(1)$ processes where there is no mean reversion, shocks in an $I(d)$ time series with $0 < d < 1$ only dissipate at a slow hyperbolic rate over time (Bollerslev and Mikkelsen, 1996).

**Fig. 1: Autocorrelation of Athens stock market absolute weekly returns (May 1998 to June 2010)**

As an illustration of the long-range dependence, we plot, in Figure 1, the autocorrelation up to 300 lags of the absolute weekly stock returns on the Athens stock market. We can unambiguously see that the return series exhibits significant autocorrelations, even for more than 280 weeks.

Empirical studies focusing on long memory are particularly interested in the modeling of non-stationary second moments of the logarithmic returns of financial assets. Ding et al. (1993) show that the absolute and squared stock returns and their power transformations are highly autocorrelated over time, even though stock returns exhibit little serial correlation. Baillie et al. (1996) and Bollerslev and Mikkelsen (1996) estimate a Fractionally Integrated GARCH process (FIGARCH) for exchange rates and for stock prices, respectively. The evidence of long memory in stock returns and exchange rates was also found in, among others, Crato and De Lima (1994), Cheung and Lai (2001), and Chkili et al. (2012).
On the other hand, the Hurst’s exponent is also commonly regarded as a traditional test for long memory in time series. This technique was introduced by Hurst (1951), based on the development of Brownian motion processes. In finance, the Hurst’s exponent has been intensively employed in empirical works to detect the long memory properties of financial markets. One of the main implications is that the market is informationally efficient if long memory is indeed present in asset returns (Mandelbrot, 1969; Fama, 1970), i.e., it is possible for market participants to gain abnormal returns by exploring the long memory patterns.\(^2\)

However, the above-mentioned approaches are not efficient given that the detection of long memory requires econometric techniques that allow for a junction between the spectral and long-lag autocorrelation definitions of long memory, as noted by Diebold and Inoue (2001). In this context, the wavelet-based MRA appears as a prominent approach, which is also used in this study together with the Hurst’s exponent. Compared with other traditional models, the wavelet-based MRA is more advantageous at least for three raisons. First, it enables to produce a complete time-scale (or time-frequency) representation of the statistical properties of a time series and can successfully be extended to pattern recognition and crash detection (Mallat and Hwang, 1992). Second, the wavelet-based MRA allows for multifractal analysis of asset returns with proven advantages to standard approaches using raw data. Finally, the multifractal model of asset returns can describe important empirical regularities observed in financial time series, including their time-scale dependent behavior and the occurrence of extreme events.

### 3. A brief review of literature

We briefly review, in this section, the major studies that investigate the long-range dependence using either the Hurst’s exponent or the wavelet-based MRA or a combination of both approaches. To start, Lipka and Los (2002) measure the degree of persistence over time in the volatility behavior for eight European stock markets through computing the Hurst’s exponent from the MRA. They show that the stock markets under consideration exhibit long-range dependencies. Di Matteo et al. (2003) analyze some scaling properties of foreign exchange rates and stock market indices using the Hurst’s exponent and find a link between market development phases and scaling exponents. Mulligan (2004) employs several empirical techniques including the Hurst’s exponent, the power spectral density and the MRA to analyze the persistence behavior of technology equity prices and finds evidence to support the multifractal

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model of asset returns, which invalidates the efficient market hypothesis (EMH). Indeed, the presence of an anti-persistent behavior in many equity prices implies the mispricing of all technology stocks. Several studies have examined the long memory in return and volatility of stock markets in specific countries (Assaf and Calvalcante, 2005; Norouzzadeh and Jafari, 2005; DiSario et al., 2008) and find evidence to support this hypothesis.

Jagric et al. (2005) use a wavelet analysis and a rolling approach of the Hurst’s exponent to examine the EMH for six transition economies and find strong evidence of long-range dependence in four out of six stock markets. In and Kim (2006) compute the optimal hedge ratios for portfolios of Australian stocks and stock futures through decomposing the raw data in various time scales and estimating the global Hurst’s exponent. Their results indicate an anti-persistent behavior of both markets under consideration. Karuppiah and Los (2005) show the existence of long-run dependence in seven Asian foreign exchange markets based on the application of the MRA to high frequency data during the 1997 financial crisis. Los and Yu (2008) analyze the persistence properties of the Chinese stock markets by means of the global Hurst’s exponent and the MRA around regulations and deregulation measures. They document that Chinese stock markets were considerably more persistent before deregulation and that they become more efficient over time. Shifting attention to the Latin American stock markets and foreign exchange markets, Kyaw et al. (2006) find that daily returns are non-normal, non-stationary and non-ergodic, and that they also exhibit long-range dependence.

Discrete wavelet decompositions have also been applied to the investigation of cross-market interactions and output dynamics. For example, Lee (2004) uses the wavelet analysis to investigate the international comovement of stock markets and reveals the impact of multiscale price and volatility spillovers from mature stock markets to emerging countries. Shar-kasi et al. (2005) employ the discrete wavelet transformation to examine the reaction of both emerging and developed markets to crashes and extreme events, and show that these markets respond to crashes differently as emerging markets may take more time to recover. Gallegati and Gallegati (2007) use the maximal overlap discrete wavelet transform (MODWT) to analyze the interactions between the industrial production indices of G7 countries on a scale-by-scale basis. The results of their analysis indicate no evidence of an international pattern of moderation in output volatility, but different correlation patterns at different time-scale components. The output linkages between countries are found to be mostly significant at the business cycle time scales. In a related study, Gallegati (2008) combines the cross correlation functions and the MODWT to investigate the interactions between stock markets and eco-
nomic activity in the United States during the period 1961-2006. The author finds that market returns lead the level of economic activity, but only at the highest scales (lowest frequencies) corresponding to periods of 16 months and longer.

More recent contributions using wavelet approach focus on the volatility spillovers in both time and frequency domains (e.g., Rua and Nunes, 2009; Maharaj et al., 2011; Madanelo and Piho, 2012). For instance, Rua and Nunes (2009) examine the volatility transmission between major developed stock markets in the time-frequency space and find evidence of volatility spillovers across stock markets simultaneously in the time and the frequency domain. Maharaj et al. (2011) use discrete wavelet decomposition to investigate the dynamics of daily volatility for a large sample of emerging and developed markets at different time scales. They find that these markets have the same volatility behavior in terms of time-scale dependence, but emerging markets exhibit higher level of volatility.

Overall, the related literature recognizes the relevance and usefulness of the Hurst’s exponent and the wavelet-based MRA in capturing the true patterns of financial return dynamics. Our study combines the global Hurst’s exponent and the wavelet-based MRA to evaluate the degree of long-range dependence in Mediterranean stock markets. This combination allows us to examine the multifractal behavior of stock returns over different time scales, which is likely to occur given that stock markets are made up of various types of investors with heterogeneous investment horizons and feature stochastic transaction costs.

4. The empirical assessment of long-range dependence

This section shows how the results from the wavelet-based MRA can be used to estimate the long-range dependence. We begin with presenting the wavelet analysis and then introduce the Hurst’s exponent approach.

4.1 Wavelet-based MRA

It is now common that the wavelet-based MRA is able to produce accurate power spectra and to accurately describe the underlying patterns of the financial data. It particularly enables us to identify the precise timing and power of the innovations or shocks occurring in the financial markets via the scalogram and scalegram. Indeed, a scalogram assesses all power spectra localized at different scales and various time periods, while the scalegram is a time-averaged scalogram. The scalograms are useful for detecting periodicities and the most aperiodic cyclicities such as trading holidays and, in our study, they are obtained by estimating the wave-
let resonance coefficients which correlate a wavelet $\psi_{j,n}$ and the return on a stock market index $x(t)$, such as

$$d_{j,n} = \int_{-\infty}^{+\infty} x(t) \psi_{j,n}(.) \, dt$$

(1)

where $j, n \in \mathbb{Z}$ and the wavelet function in our empirical investigation, $\psi(.)$, is represented by the Morlet wavelet. The latter is a localized function of the following form

$$\psi_{j,n}(.) = \pi^{(-\frac{1}{2})} e^{i\omega_0 n} e^{(-\frac{n^2}{2})}$$

(2)

where $\omega_0$ is the fundamental angular frequency (or the wave number), $\omega_0 \sim \frac{2\pi}{n}$, and $n$ is the number of observations.

The variance of the zero-mean wavelet resonance coefficients, $d_{j,n}$, can be calculated as follows

$$\text{Var}(d_{j,n}) = E \left\{ |d_{j,n}|^2 \right\}$$

(3)

Eq. (3) gives a simple variance scaling law from which the Hurst’s exponent can be directly derived from the scaling slope.\(^3\)

4.2 Long-range dependence and Hurst’s exponent

The Hurst’s exponent is mutually related to the Fractal Brownian Motion (FBM) according to which the return series is viewed as an affine process with fractionally differenced white noise. This kind of process has been extensively studied by Mandelbrot and Van Ness (1968), Granger and Joyeux (1980), Hosking (1981) and Sowell (1990).

On the practical side, the Hurst’s exponent can be estimated using the rescaled range (R/S) analysis. Formally, the R/S statistic ($R/S_H$) is computed as follows

$$(R/S_H(T)) \equiv \frac{1}{c_2^{0.5}} \left\{ \max_{1 \leq t \leq T} \sum_{t=1}^{T} [x(t) - m_1] - \min_{1 \leq t \leq T} \sum_{t=1}^{T} [x(t) - m_1] \right\} \geq 0$$

(4)

where

$$c_2 = \frac{1}{T} \sum_{t=1}^{T} [x(t) - m_1]^2$$

(5)

and $x(t)$ is return on a stock market index as stated previously.

Then, the Hurst’s exponent $H$ is defined as

$$H = \lim_{T \to \infty} \frac{\ln \left( \frac{R/S_H(T)}{S_H} \right)}{\ln(T)}$$

(6)

\(^3\) See Los (2003) for more technical details regarding the Morlet wavelet.
In connection with the wavelet-based MRA approach, Kyaw et al. (2006) show that the Hurst’s exponent can be derived from the logarithmic plot of the power spectrum \( P(\omega) \) of the FBM with respect to the logarithm of frequency \( \omega \), since

\[
\ln P(\omega) = -(2H + 1) \ln \omega + \ln \sigma^2 + \ln C
\]

where \( C \) is a proportion constant, also called pre-factor. \( \sigma^2 \) is the variance of the return series.

Alternatively, the Hurst’s exponent can also be computed from the logarithmic plot of the wavelet scalegram of the FBM with respect to the scaling level \( a \), such as

\[
\log_2 P_w(a) = (2H + 1) \log_2 a + \log_2 \sigma^2 + \log_2 D
\]

where \( D \) is a proportion constant.

Specifically, the results from the Hurst’s exponent may lead to three different diffusion processes of stock price dynamics. First, when \( H = 0.5 \), the process is said to be Fickian or neutral, which means that the return realizations are mutually independent throughout time and market operators cannot use past available information to earn abnormal returns. Second, the anti-persistent behavior is observed when \( 0 < H < 0.5 \). This implies that the return series is more ergodic or mean-reverting or more volatile than a random walk process. This is the kind of business environment which promotes competition and innovation because firms respond to the uncertain business conditions with experimental and dynamic resource allocations. Furthermore, the investment risk can be considerably reduced when highly volatile returns are uncorrelated across different assets. With reference to the well-known efficient market hypothesis (Fama, 1970), a Hurst’s exponent value significantly lower than 0.5 indicates a strong rejection of informational efficiency in the sense that market operators persistently overreact to new information, generating more price volatility than that would be consistent with the EMH. Finally, the return process is said to have a persistent behavior when \( 0.5 < H < 1 \). The higher the value of \( H \) the more the investors should earn positive returns (Mulligan, 2004) and the more stock price deviates from the equilibrium price with respect to the EMH. In this context, Neely et al. (1997) conjecture that technical trading rules, formalized with a genetic programming algorithm, may provide significant out-of-sample excess returns. When the Hurst’s exponent is close to one, there is a high probability of observing large sudden fluctuations.

The above discussions show that the long memory effect is present when the return series exhibit persistent or anti-persistent behavior. In theory, this implies that actual realiza-
tions will impact the future ones in a nonlinear manner. If a persistent price goes down (up) during the last period, then there is chance that it will continue to go up (down) in the next period (Los and Yu, 2008).

5. Data and empirical results

5.1 Data

We use weekly closing stock price indices, expressed in local currencies. The data were extracted from DataStream International. Table 1 gives a description of our dataset. Figures 2 and 3 present the evolution of stock market indices and index returns over time. It can be seen that return series are characterized by several periods of high volatility which mostly correspond to the terrorist attack on 11 September 2001 and the recent global financial crisis 2007-2009.

Table 1. Description of the dataset

<table>
<thead>
<tr>
<th>Country</th>
<th>Stock market index</th>
<th>Sample period</th>
<th>No. of observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Egypt</td>
<td>CASE-30</td>
<td>07/11/1997-06/25/2010</td>
<td>678</td>
</tr>
<tr>
<td>Tunisia</td>
<td>TUNINDEX</td>
<td>12/31/1997-06/25/2010</td>
<td>651</td>
</tr>
<tr>
<td>Turkey</td>
<td>ISE National-100</td>
<td>07/11/1997-06/25/2010</td>
<td>678</td>
</tr>
<tr>
<td>France</td>
<td>CAC40</td>
<td>07/03/1995-06/21/2010</td>
<td>782</td>
</tr>
<tr>
<td>Greece</td>
<td>ASE index</td>
<td>05/01/1998-06/25/2010</td>
<td>628</td>
</tr>
<tr>
<td>Spain</td>
<td>IBEX30</td>
<td>07/03/1995-06/21/2010</td>
<td>782</td>
</tr>
</tbody>
</table>

Fig. 2: Behavior of stock prices over time

France (CAC40)  
Greece (ASE-30)  
Spain (IBEX-30)  
Turkey (ISE-NATIONAL-100)  
Egypt (CASE-30)  
Tunisia (TUNINDEX)
Fig. 3: Time-variations in weekly stock market return

France (CAC40)  
Greece (ASE-30)  
Spain (IBEX-30)  
Turkey (ISE-NATIONAL-100)  
Egypt (CASE-30)  
Tunisia (TUNINDEX)

Table 2. Stochastic properties of the weekly returns

<table>
<thead>
<tr>
<th></th>
<th>Egypt</th>
<th>Turkey</th>
<th>Tunisia</th>
<th>Greece</th>
<th>France</th>
<th>Spain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (%)</td>
<td>0.204</td>
<td>0.491</td>
<td>0.245</td>
<td>-0.093</td>
<td>0.076</td>
<td>0.135</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.733</td>
<td>-0.193</td>
<td>0.979</td>
<td>-0.264</td>
<td>-0.824</td>
<td>-0.938</td>
</tr>
<tr>
<td>JB</td>
<td>54.13***</td>
<td>34.67***</td>
<td>41.09***</td>
<td>36.43***</td>
<td>23.88***</td>
<td>27.65***</td>
</tr>
</tbody>
</table>

Notes: this table shows descriptive statistics for weekly returns in percentage. JB is the empirical statistics of the Jarque-Bera test for normality. *** indicates rejection of the null hypothesis of normality at the 1% level.

We compute the weekly returns as \( r_t = 100 \times \ln(P_t/P_{t-1}) \), where \( P_t \) is the index price at time \( t \). Table 2 reports descriptive statistics for the return series. Average weekly re-
turns are positive for all markets, except for Greece. Turkey has the highest average return of 0.491%, followed by far by Tunisia (0.245%) and Egypt (0.204%). All the return series exhibit negative skewness, except the Tunisian stock market. The kurtosis coefficients are all above 3 for all the series, except for Turkey. These findings suggest that the probability distributions of the considered return series are asymmetric and exhibit leptokurtic (fat-tailed) behavior, as compared to the normal distributions. The rejection of the normality is clearly confirmed by the Jarque-Bera test as well as the quantile-quantile (QQ) plots (Figure 4). Indeed, the QQ plots show evidence of return deviations from the normal distribution (red line) and the presence of extreme return values.

**Fig. 4: The Quantile-Quantile plot for the weekly stock returns**

<table>
<thead>
<tr>
<th>Country</th>
<th>Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
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<tr>
<td>Tunisia</td>
<td>TUNINDEX</td>
</tr>
</tbody>
</table>

5.2 Scalograms and scalegrams from the wavelet-based MRA

We now perform a country-by-country analysis of the scalograms and scalegrams that we obtain from the wavelet-based MRA. We present, for each market under consideration, a figure containing three different graphs. The graph (a) reproduces the original return series together...
with the selected wavelet (the Morlet 6.0). The graph (b) or the scalogram provides the localized power spectrum which corresponds to a colorized plot of the squared value of the wavelet resonance coefficients (i.e. the time-frequency-localized coefficients of determination). The graph (c) or the scalegram shows the global wavelet which plots the variances of the zero-mean wavelet against the time scales. It can also be viewed as the statistical time average of the scalogram. It is worth noting that we implement the Kodak’s online ION Script Research Systems Interactive Wavelet Program to compute the wavelet resonance coefficients by Mallat’s (1989) wavelet-based MRA for each selected country.

According to Los (2003), c is the slope of the scalegram of the stock price level and is closely connected to Hurst exponent since $c = 2H + 1$, and the Hurst’s exponent is equal to $H = (c - 1)/2$. Similarly, the slope of the scalegram of stock returns is $d = 2H - 1$, implying that $H = (d - 1)/2$. Los (2003) empirically shows that the scalograms provide the identical value of the Hurst’s exponent. In this study, the Mallat’s (1989) wavelet-based MRA scalogram is used to compute the wavelet resonance coefficients. As in previous studies (e.g., Kyaw and Los, 2006; Los and Yu, 2008), we use the Morlet 6.0 wavelet to decompose the return series because it allows the precise identification of the timing and the power of return innovations or volatility shocks in the selected stock markets.

5.2.1 Egypt

The wavelet analysis for the Egyptian stock market shows the presence of three main striking vortexes whose change quickly from the low to high frequency. The most important vortex corresponds to the second week of November 2006. It covers most of the frequencies with considerable power. This stylized fact is confirmed by the plot of the weekly logarithmic stock return in the graph (a) of Figure 5. Indeed, the CASE-30 index had a sudden decrease from 2,218.24 points to 1,687.65 points over one week and then rose up to reach 2,284.65 points one week later. The scalogram also indicates more extreme return movements since 2005 at both low and high frequencies, which probably result from the weakened economic policies and the lack of confidence on economic stability (Askar et al., 2006) as well as from unfavorable conditions in international monetary markets following the US Federal Reserve’s decision to raise the interest rate to 5.25%. Before this, there is a relatively stable period

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4 A Morlet-6 wavelet is advantageous in that it can precisely capture a non-smooth asymmetric distribution by means of six non-vanishing moments (Kyaw et al., 2006). By contrast, a smooth and symmetric normal distribution has only two unique non-vanishing moments.
marked by the positive effects of financial policy reforms which aim to attract investments from both local and international investors.

The second striking vortex in red color (i.e., highest power) is observed at the 588th weekly observation and seems to reflect the significant fluctuation of the Greek market during the US subprime crisis. Indeed, during the first week of October 2008, the CASE-30 index decreased by 14.14% from 2354 points to less than 2,021 points. The last turbulent period is marked by the continuation of sharp movements at low frequencies from the previous period, but the amplitude is lower. There are also some significant changes stating on June 2009, indicated by smaller vortexes at high frequencies.

**Fig. 5: Weekly stock return, wavelet power spectrum and the global wavelet for Egypt**

We use weekly data for the CASE-30 index from January 1997 to June 2010. The graph (a) reports the original return series and the type of wavelet used in our analysis (Morlet 6.0). The graph (b) is the scalogram which is a colorized plot of the magnitude of the wavelet resonance coefficients. The graph (c) is the scalegram which is the equivalent of the logarithm of the power spectrum, i.e., the Fourier transform of the series’ autocorrelation function (ACF).

![Wavelet Analysis](image)

5.2.2 Turkey

The results from wavelet analysis for this market is presented in Figure 6. The graph (a) suggests that the ISE National-100 index has had various spikes over the study period. The Turkish market was also affected strongly by the Asian financial crisis as it experienced a very large negative return during October 1997. The stock price dropped from 3,512 points on October 24, 1997 to 2,846 points by the end of the same month. The most significant price change is however observed during the period 2000-2001 which coincides with the Turkish financial crisis. The ISE National-100 index dropped from 19,110 points in January 2000 to less than half at 8,022 points in March 2001. The 2008-2009 global financial crisis also af-
fected severely the Turkish stock market as it lost more than the half of its index value between October 2007 (58,053 points) and December 2008 (26,499 points).

The plot of the wavelet spectrum power in the graph (b) shows important changes in the time-path of the Turkish stock market returns with respect to time horizons and frequencies. We see some considerable movements of returns at the high and medium frequencies (from 2 to 16 weeks) during the Asian financial crisis. The 2000-2001 Turkish financial crisis is illustrated by striking red and orange vortexes. The latter indicate the first period of extreme movements going from relatively low to high frequency. The second turbulent period is marked by the intense reaction of the Turkish market to the 2008-2009 global financial crises, but the change in return series is less important than the first turbulent period, in view of the smaller red and orange vortex.

**Fig. 6: Weekly stock return, wavelet power spectrum and the global wavelet for Turkey**

We use weekly data from January 1997 to June 2010. See also the legend of Figure 5 for detailed explanations.

5.2.3 Tunisia

The graphs (a) and (b) of Figure 7 reveal various shocks to the weekly returns on Tunisian stock market index. Similar to other MENA countries, the Tunisian stock market was negatively affected by some extreme events such as the September 11 terrorist attack (a decrease of 4.5% in the TUNINDEX) and the 2008 subprime crisis (a decrease of 6.75% in the TUNINDEX) in the United States. The red and orange vortexes are located at low, medium and high frequencies. It is worth noting that the extreme movements are plenty frequent under the effects of global economic uncertainty that are induced by the recent global financial crisis 2008-2009. For example, the scalogram detects an extremely negative return of 6.75% at the 564th observation which corresponds to the US stock market crash in the second week of Oc-
October 2008, marked by the worst loss for the S&P 500 index (21.6%) between October 1 and October 9. On the contrary, the dynamics of the TUNINDEX experiences some upward jumps during the beginning of January 2000 (+8.34%) and the first week of March 2008 (+7.91%).

Fig. 7: Weekly stock return, wavelet power spectrum and the global wavelet for Tunisia
We use weekly data for the TUNINDEX from January 1997 to June 2010. See the legend of Figure 5 for detailed explanations.

5.2.4 France

The wavelet-based scalogram in Figure 8 detects several sudden changes in the time-path of CAC 40 index returns, which are displayed in the graph (a) and correspond to the major world market events. The main extreme movements occurred in October 1998 at the 169th weekly observation, in September 2001 at the 323rd weekly observation, in October 2008 at the 692th weekly observation, and in April 2010 at the 774th weekly observation. For example, the CAC 40 index lost about 900 points (from 4,080 points to less than 3,180 points) or 22.05% during the first week of October 2008 under the severe effects of the US subprime crisis.

Fig. 8: Weekly stock return, wavelet power spectrum and the global wavelet for France
We use weekly data for the CAC 40 index from March 1995 to June 2010. See the legend of Figure 5 for detailed explanations.
The French stock market was also particularly sensitive to the Greek economic and public debt crisis as well as to its contagious effects towards Portugal and Spain. Some French leading banks had to undertake the debt restructuring operations. Lastly, the time-scale behavior of the French stock market is somewhat similar to that of Spain because it contains some extreme movements at high frequency and consistent wide variability throughout the study period. This pattern of extreme changes is reflected by various red vortexes in the top and a long-lasting red vortex in the bottom of graph (b).

5.2.5 Greece

Stock returns in Greece show strong fluctuations during 1998 following the Asian financial crisis, the September 11 terrorist attack, the 2008 subprime crisis and the Greek public debt crisis. With respect to the scalogram in graph (b), stock market crashes can be easily identified by the sudden spikes in power or the migration of blue, green to red color. By the end of April 2010 (620th weekly observation), we can detect the burst of higher power which spreads from high frequencies to low frequencies. This episode of extreme movements was sparked off, on 23 April 2010, by the fact that the Greek government required an initial loan of US $61 billion from the European Union and the International Monetary Fund (IMF) bailout package to cover its financial needs for the remaining part of 2010. Furthermore, the Standard and Poor’s downgraded the sovereign debt quality of Greece to BB+ on 27 April 2010 and expressed serious fears of default risk of the Greek government. The Greek and other European stock markets significantly declined in response to this downgrade announcement.

Fig. 9: Weekly stock return, wavelet power spectrum and the global wavelet for Greece
We use weekly data for the ASE index from January 1998 to June 2010. See the legend of Figure 5 for detailed explanations.
5.2.6 Spain

The graph (a) in Figure 10 shows that the dynamics of the IBEX 35 index exhibits three large changes over the period 1995-2010. This pattern is also confirmed by the wavelet-based scalogram in view of the three red and orange vortexes at medium and high frequencies. The latter typically indicate significant declines in the value of the IBEX 35 index following the Asian financial crisis (from 7,969 points on September 28, 1998 to 7,156 points on September 5, 1998), at the time of the September 11 terrorist attack (15.60% loss), and during the first week of October 2008. Beside these extreme movements, the scalogram also displays some fluctuations of smaller magnitude such as the reaction of the IBEX 35 index to terrorist incident which occurred on March 11, 2004 in Madrid as well as to the Spanish government’s announcement of new austerity measures, under the pressure of the IMF and the European Commission, in order to reduce the country’s budget deficit (775th weekly observation, May 2010). Notice that the Spanish stock market is highly volatile over the long term (between 256 and 512 weeks) as a red and orange vortex goes through the study period.

**Fig. 10:** Weekly stock return, wavelet power spectrum and the global wavelet for Spain

We use weekly data for the IBEX 35 index from March 1995 to June 2010. See the legend of Figure 5 for detailed explanations.
Overall, our results show that the wavelet-based MRA is a powerful approach to detecting specific movements in stock market returns across both time and frequency spaces. It is particularly useful for capturing the power (amplitude) of these return movements through the analysis of colored scalogram. Or studied markets generally experienced wide and extreme movements in response to important shocks that affect the world financial system such as the Asian financial crisis, the September 11, 2001 terrorist attack, the global financial crisis and the European debt crisis. Notice that the wavelet-based MRA reveals the high degree of long-run volatility in two developed markets, France and Spain. This finding can be explained by the high level of global market integration of these two markets, implying that they are more exposed to international financial shocks.

5.4 Hurst’s exponents and persistent behavior of stock markets

As stated earlier, the global Hurst’s exponents assess the degree and the nature of long-term dependence in stock market returns. In this study, we compute, for each stock market under consideration, the global Hurst’s exponent using the time-averaged variances of the zero-mean wavelet resonance coefficients (i.e., the scalegrams of the Figures 5-10). The obtained results, reported in Table 3, show that we can broadly distinguish two groups of markets according to their long-term return behavior. The first group, which is composed of Egypt, Greece, Spain, Tunisia and Turkey, is characterized by a long-range memory (i.e., persistent behavior). The Hurst’s exponent ranges from 0.52 (Spain) to 0.69 (Tunisia). The Spanish and Turkish markets are only weakly persistent given that the associated Hurst’s exponents are very close to 0.5. For this group of markets, it is clear that the weak-form market efficiency in the sense of Fama (1970) cannot be accepted and market participants may generate abnormal
profits by building long memory based portfolios. The actual market trend serves, in this case, as important information for portfolio allocation decisions.

Another group is formed by the French stock market only, whose return dynamics is governed by an anti-persistent behavior. Effectively, the Hurst’s exponent for this market is lower than 0.5 and this anti-persistent dynamics implies that stock returns are ultra-fast mean-reverting. Our results thus invalidate the weak-form market efficiency for the French market because market operators tend to persistently overreact to new information, generating more price volatility than it would be consistent with the efficient market state.

Table 3. Global Hurst’s exponents

<table>
<thead>
<tr>
<th>Country</th>
<th>Hurst’s exponent</th>
<th>Fractional differencing parameter (d)</th>
<th>Market dependence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Greece (ASE index)</td>
<td>0.67</td>
<td>1.17</td>
<td>Persistent</td>
</tr>
<tr>
<td>Egypt (CASE 30 index)</td>
<td>0.60</td>
<td>1.10</td>
<td>Persistent</td>
</tr>
<tr>
<td>Spain (IBEX 30 index)</td>
<td>0.52</td>
<td>1.02</td>
<td>Weak-persistent</td>
</tr>
<tr>
<td>Tunisia (TUNINDEX)</td>
<td>0.69</td>
<td>1.19</td>
<td>Persistent</td>
</tr>
<tr>
<td>Turkey (ISE NATIONAL 100 index)</td>
<td>0.56</td>
<td>1.06</td>
<td>Weak-persistent</td>
</tr>
<tr>
<td>France (CAC40)</td>
<td>0.48</td>
<td>0.98</td>
<td>Anti-persistent</td>
</tr>
</tbody>
</table>

Notes: The global sample period is running from January 1995 to June 2010. Stock index prices are weekly and the number of observations varies across countries. Wavelet MRA is employed to generate the global Hurst’s exponents. We should recall that when the Hurst’s exponent is equal to 0.5, the stock price returns dynamics are governed by a Geometric Brownian Motion (GBM) with white noise. This implies that the stock market behavior is consistent with the EMH and there’s no long dependence structure. When 0 < H < 0.5, the stock market exhibit anti-persistent behavior over time, implying that the stock market is superefficient or fast mean-reverting and has, thus, non-neutral memory. However, when 0.5 < H < 1, the stock market dynamic is persistent, implying that the nodal market is sub-efficient and only slowly reverts to its moving mean and subsequently, it has non-neutral memory. In this table, we also report the fractional differencing parameter d of where H = d + 0.5.

Recall that our Hurst’s exponents represent the time-averaged behavior of stock markets. Put it differently, they may have quite different values for various subsamples or estimation windows. For example, Cajueiro and Tabak (2004) estimate the Hurst’s exponent for a large sample of emerging stock markets over a four-year window and find evidence to support the time-varying hypothesis of long-range dependence. Using the rolling-window approach of Cajueiro and Tabak (2004, 2008), Aloui and Hamida (2011) find evidence of persistent behavior for the Tunisian stock market, which is consistent with our result. The weak long-range dependence in Turkish stock market returns was also documented in Gursakal et al. (2009)’s study. Our results are however more robust than those of the previous studies because the wavelet-based MRA enables us to account for the presence of extreme events and nonlinear patterns in stock price developments with respect to both time and frequency domains.

Overall, our study shows that selected emerging stock markets have stronger persistence behavior than selected developed markets. The lack of adequate infrastructure and legal environment (fiscal policy and legal protection of minority shareholders) as well as the poor quality of information disclosure are important sources of the strong persistent behavior
which leads to the rejection of the EMH. Notice however that most of these institutional dysfunctions and inadequacies have been progressively reduced in recent years. An analysis of the Hurst’s exponent over time will certainly provide valuable insights about the gradual impacts of these ongoing changes on stock market efficiency. The result of our empirical investigation is consistent with the findings of Jagric et al. (2005) and Cajueiro and Tabak (2008). Using the traditional rescaled range (R/S) and rescaled variance (V/S) methods to investigate long-term dependence for a large sample of emerging and developed stock markets, these authors provide evidence that emerging market returns exhibit stronger long-range dependence than developed market returns.

6. Summary and concluding remarks

An important body of the empirical finance literature has been devoted to the investigation of the long-range dependence in financial markets. In this article we combine the wavelet-based MRA analysis with the mono-fractal global Hurst’s exponent to examine the behavior of six Mediterranean stock markets, of which four are emerging and two are developed. This empirical approach is not only useful for the detection of extreme movements in return series over time, but also for the assessment of the degree of their long-range persistence. Using weekly data over the recent period, our results reveal that the four emerging markets and the Spanish market exhibit persistent behavior with the associated Hurst’s exponents greater than 0.5, suggesting that these markets are not informationally efficient in the sense of the weak-form market hypothesis (Fama, 1970). However, the Spanish and Turkish markets are only weakly persistent as the Hurst’s exponents are very close to 0.5. Moreover, the French stock market is found to have an anti-persistent behavior, which implies that stock returns are governed by a very fast mean-reverting process. Finally, the wavelet-based MRA is found to be flexible enough to describe the overall dynamic patterns in stock returns of considered markets and also to capture various extreme reactions of stock markets to major events of the world’s financial system such as the Asian financial crisis, the September 11, 2001 terrorist attack and the 2008 global financial crisis.

Our empirical findings have several important implications for both market participants and policymakers. On the one hand, in a non-Fickian or non-neutral market, return persistence systematically implies the existence of profit-making arbitrage opportunities, making the considered market inefficient and unfair. Policymakers thus have interest to undertake appropriate actions to improve the efficiency of stock markets in order to ensure their attractiveness as
a long-run financing source for promoting economic growth. Measures such as the implementation of adequate accounting standards, the enhancement of disclosure requirements, and the modernization of the trading and settlement systems are among the most important reforms that permit to reduce stock market imperfections. On the other hand, the weak persistence detected for some stock markets (Turkey and Spain) indicates that it is nearly not possible to make abnormal profits from the past information contained in stock returns. Future research may apply our empirical approach to consider the daily and intraday data since the latter are likely to exhibit more extreme movements than weekly data.

References


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