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Cyclical components and dual long memory in the foreign exchange rate dynamics: the Tunisian case

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Abstract
The purpose of this paper is to question the traditional conventional view on the exchange rate targeting that real shocks have permanent effect on exchange rates (FX) however nominal shocks are not. Thus, an empirical approach is proposed in order to analyze the transitory component dynamics of some major Tunisian interbank FX rates for the period 1999-2005. Our results reveal that the use of the Guy and Amant’s (2005) method allows us to select the Hodrick Prescott with two powers as an optimal filter for extracting the daily interbank FX rates’ cyclical components. More importantly, the joint estimations of an ARFIMA model in the mean equation and various long-memory GARCH-type models in the variance equations reveal that cyclical components seem to be well described by dual long memory models. On the practical side, our findings provide important evidence that transitory trend fluctuations are not quickly trend-reverting but they are rather dominated by permanent deviations from the equilibrium values. Accordingly, contrary to policy makers’ ambitions for the Tunisian dinar, our study appears to confirm the view that monetary shocks may also (as for real shocks) be a difficult task of stabilization policy. This result may have several important implications for monetary policy in most developing countries.

Keywords: exchange rates; time series decomposition; HML test; dual long memory.

JEL classification: E30, F31
1. Introduction

Over the last four decades, the foreign exchange (FX) rates in small-open economies have shown substantial fluctuations which are generally unrelated to macroeconomic fundamentals. Subsequently, it is crucially important for small-open economies’ authorities to identify the sources of FX rate volatility and achieve successful FX stabilization. For academicians, FX rate movements have been subjected to a very intensive debate. One of the most frequently debated topics is to check whether FX rate dynamics are really governed by fundamental economic forces or by some behavioral aspects of the participants in FX market. To provide more explanations to FX rate dynamics, there has been widespread resort to theoretical and empirical models in which the presence of unit root, permanent and transitory components play a key role in the nominal FX behavior. In this sense, Whitt (1992) noted, “The presence of a unit root in a variable means that it is subject to permanent stochastic shocks, not merely temporary shocks around a deterministic level or trend” (Whitt 1992, p. 539).

In their study, Engel and Kim (1999) showed that monthly USD/GBP real exchange rate is found to be nonstationary. Using the univariate Kalman filter model, the authors showed that the USD/GBP FX rate dynamics can be decomposed into transitory and permanent components. While the transitory component is closely related to monetary factors, the permanent component is linked to the per capita output levels. The analysis is consistent with the classical dichotomy between the roles of real and monetary shocks: real shocks have a permanent effect on real exchange rate, whereas monetary (nominal) shocks have only a temporal effect.

Contrary to this latter dichotomy, Evans and Lothian (1993), in particular found that real changes can generate purely temporary effects. On the other hand, there are some theoretical reasons to suggest that monetary changes may exert long lasting real effects (through inter-temporal smoothing of traded goods consumption as proved by Rogoff(1992), or cross country wealth redistribution effects as suggested by Obstfield and Rogoff, (1995a)). Consequently, the analysis of the memory structures of real and monetary shocks and their behaviors, which is a widely neglected dimension in the empirical literature, should remain open as an empirical issue.

Andersen and Bier (2005) examined the transmission of monetary shocks in new open economy models and underlined the need to distinguish between transitory and persistent changes. They discussed the key role the information arrivals plays to understand the FX market volatility since they bring about an instantaneous exchange rate response. They also demonstrated that the Eurodollar exchange rate (as an example) exhibits
highly persistent errors in expectations indicating fundamental information problem. Interestingly, the authors argue that the highly persistent departures of the real exchange rate from its expected level do not necessarily imply market anomalies but they may originate from the non-trivial difficulty of perfectly distinguishing transitory from permanent shocks. In sum, a major finding is that imperfect information and the accumulation of information over time, i.e., learning can have significant implications for the dynamic adjustment path to shocks and its additional persistence respectively.

The main purpose of this study is to specify the nature of the observed trend process in the nominal FX behavior. We throw the required insight by studying the memory structure of the cyclical component trend. Our empirical investigation is focused on the Tunisian interbank FX market. On the practical side, we ignore the effects of the permanent component in order to exclusively focus on the dynamic of that portion of FX rate volatility due to monetary policy shocks. The perception that cyclical components can spend long period away from their equilibrium level implies a revival of interest of the previous studies on the nature FX rate deviations from their equilibrium level.

Indeed, policy implications are really important. In the case of temporary disequilibrium, there might be some room for the policy makers to prevent further disequilibrium through taking accurate policy measures that correct excessive imbalances and promote fast convergence towards the equilibrium level. Conversely, in situations of prolonged deviations from the equilibrium level, efforts towards more rigorous stance in macroeconomic policies and increased convergence would prove futile (Dufrénot et al., p. 2, 2008). Hence the question whether transitory/permanent components do or do not exhibit long-range dependence has significant consequences for the stabilization policies. The vast majority of existing empirical studies attempt to investigate the importance of permanent and transitory shocks in explaining the FX rate volatility for various developed countries.

Ciner (2011) investigated the transmission of information between the currency future markets and identified strong informational dependencies between the euro, yen, Swiss franc and pound. He tested for permanent and transitory dependencies by decomposing the information content of causality analysis: long term transmissions of information are identified via dependency tests at near-zero frequency of the spectra (permanent shock) whereas short term linkages are determined through dependency tests at higher frequencies (transitory shock).
shocks). He claims that testing for informational linkages at different frequencies can produce richer and more precise dynamic analysis (see also, Narayan, 2008 among others).

Nevertheless, empirical evidences on this specific topic have remained very limited from most of the emerging countries (we cite the two studies by Chen and Wu (1997), Ahmad and Pentecost (2009) who provide some empirical evidences on this issue for four pacific basin countries and nine African countries respectively). With reference to Clarida and Gali’s (1994) framework and for the case of Tunisia, Daly (2006) estimated a three dimensional version of structural vector autoregressive (VAR) model for the Tunisian FX market. The authors have decomposed shocks into three categories: supply, demand and monetary. Their results reveal that real shocks play a crucial role in determining the real FX rate behavior. In this light, studies apprehending the importance of permanent and transitory shocks in explaining exchange rates appear to be very helpful in forecasting FX rate behavior and guiding the monetary authorities’ decision making.

In this sense, using fractional integration technique, Gil-Alana (2006) explored the long memory properties of the Japanese real effective exchange rate by examining simultaneously the long run (or zero frequency) and the seasonal structures of the series. He found evidence of higher order of integration at the long-run or zero frequency than the seasonal one. So, he argued that shocks affecting the long run structure of the series are permanent and contrary to the seasonal one, more active policy actions are required to bring the series back to its original long-term projection. Similar behavior patterns have also been observed in other economic aggregates such as unemployment rate (Alana, 2005) and stock market returns (Caporale and Alana, 2007).

More recently, Lu and Guegan (2011) used Robinson’s (1994) method to test the presence of unit root and long run dependence of 23 FX rates. They confirmed that their results are very helpful in understanding exchange rates’ movements especially for countries that maintain flexible exchange rates and under accelerated integration of financial systems.

Based on a more general class of fractional integrated models, Caporale and Alana (2010) focused on modeling and forecasting long memory in the volatility of exchange rates particularly the US dollar against the Euro and the Japanese Yen. Their results show that the US dollar-Euro exchange rate seems to be well described by cyclical long memory model while the standard I(d) model appears to be appropriate for the US Dollar-Yen.
Harris et al. (2011) tried to extract the cyclical components of the intraday range of GBP/USD, JPY/USD and CHF/USD exchange rates (decomposed via the Hodrick Prescott and Christiano-Fidzgerald (CF) filters) in order to investigate their long run predictabilities. Accordingly, they compared the ability of the cyclical volatility model to forecast over the range-based on EGARCH and FIEGARCH models. The out-of-sample results show noticeable improvement and superior forecasting ability when using the proposed model.

At this point, given that the issue of whether deviations from exchange rates are transitory or permanent has been the focus of much recent works (Caporale and Alana (2010)), we try to direct attention away from the question of the quantitative importance of transitory shocks in determining the FX rate behavior, and toward the question of their persistence.

Thus, our research is inspired by the work of Harris et al. (2011) and completed with an investigation of the memory properties of the cyclical trend extracted from daily FX rate series. More specifically, this paper is concerned with the practical application of the Hodrick Prescott (HP) and Baxter King (BK) filters to daily interbank FX rates. To extract the cyclical movement of the series in an efficient way, it is worthwhile to consider the claim of Uebele and Ritschl (2009) who affirm that the transformed stationary signals may give misleading results since the proposed detrending procedure may misrepresent the frequency content of the remaining cyclical component.

Based on the reasoning outlined above, the kind of spectral analysis used is the decisive factor in determining the data filtering accuracy. For that reason, we rely on an efficient procedure which is proposed by Guy and St Amant (2005) to evaluate the performance of three filter techniques in extracting the more accurate estimates of exchange rate’s output gaps. Many recent studies have questioned the reliability of different filtering methods used in different fields of study especially the most widely used namely; HP filter. In this sense, Uebele and Ritschl (2009) use three detrending methods namely HP, BK and CF in order to capture the cyclical components of German incomes, taxes and expenditures. They find that for the cycles around HP trend, it is evident that the nominal series exhibit much clearer behavior than the deflated one.
Metz (2009) extended the work of Uebele and Ritschl (2009) by attempting to evaluate the performance of the HP filter in isolating both trend and cycles of German NNP income series. They argued that the cyclical components produced by this filter may be disturbed by irregular variations since it is an approximation of a high-pass filter instead of a band-pass filter. In the same way, Ahamada and Jolivaldt (2010) conducted a simulation on the American GDP using HP and BK techniques to extract its cyclical component. In addition, they compared the performance of these two popular approaches to the wavelet filtering method. They concluded that the two filters are less powerful compared to the wavelet although the three methods have comparable performances overall.

In their work, Perron and Wada (2009) concentrated on the Beveridge-Nelson (BN) and unobserved components (UC) decomposition methods and compared their performances to the HP and BK filters in extracting the cycles of the US real GDP. The interesting result is that the decomposition with HP filter seems to be more robust. They noted that the latter result depends crucially on the choice of the smoothing parameter and that larger values for this parameter (800000 in their case study) lead considerably to better cycle’s extraction. Perron and Wada (2009) claimed that the use of the default smoothing parameter of 1600 seems to be more appropriate in most cases. Their analysis adds a new major attribute by helping to optimize the use of filtering methods in the extraction of cycles. The intuition of Perron and Wada (2009) has then been adopted by Harris, Stoja and Yilmaz (2011) who set the smoothing parameter to the commonly used value of 100 multiplied by the squared frequency of the data (i.e. 5700000 for daily exchange rates).

The new insight of our paper is that we attempt to alleviate all the relevant previously identified issues via the implementation of a more efficient way to extract the daily cyclical components of Tunisian exchange rates. To address this, we use the approach of Guy and St Amant (2005). Thereby, we try to compare the filtering performance based on the spectral behavior of the signals. We also optimize the filtering process by carefully determining the best values for the smoothing parameters.

In addition, long memory and fractional integration methods have received increased attention in recent years as the power of familiar tests for unit roots are dramatically decreasing and since frictions in the foreign exchange market are present (Caporale and Alana, 2010). This paper focuses on the dual long memory aspects of the cyclical component of FX rates and highlights the importance of that component in describing the data (Harris Stoja and Yilmaz, 2011). We therefore proceed to estimate an autoregressive fractionally integrated moving
average model (ARFIMA) following Sowell (1992a)’s methodology. Finally, we estimate ARFIMA model for the FX mean dynamics jointly with two alternative long-memory GARCH-type models for the conditional variance behavior, namely the Fractionally Integrated ARCH (FIGARCH) model (Baillie et al., 1996), and the hyperbolic GARCH (HYGARCH) (Davidson, 2004) models. The last model is assimilated to a generalization of the FIGARCH model with hyperbolic convergence rates.

As mentioned above, we are concerned with the Tunisian case. Our study reveals that HP filter with two power terms performs better in spelling up these time series. More interestingly, we provide evidence of long memory entails that monetary shocks to interbank FX rates do not have a short-run transitory effect, but that they last for a long time. To the best of our knowledge, this is the first study in the Tunisian context. Moreover, it is worthy to note that our research may be considered as a case-study. However, our results have major practical implications for monetary policy-making, currency portfolio hedging and interbank FX rate forecasting in small open economies.

The rest of the paper is set out as follows; section 2 provides an overview of the Tunisian interbank FX market. Section 3 exposes the two empirical methodologies which are based on the HP (1980) filter technique and the long-memory GARCH-type models. The empirical results are displayed and discussed in section 4. Section 5 concludes the analysis.

2. Tunisian interbank FX market: a brief overview

As many emerging economies, Tunisia has experienced many institutional changes in its financial system over the last thirty years. Historically, since the collapse of the Bretton-Woods system (1973), Tunisia has adopted a fixed or intermediary exchange rate policy. However, the Tunisian foreign exchange policy can be classified into five consecutive time periods (Safra and Marzouka, 1987). During the period 1973-1978, the Tunisian Dinar (TND) was pegged to the French Franc. This era was marked by a huge instability in the foreign exchange markets which led Tunisian authorities to use the Deutschmark as a benchmark currency. From 1978 to 1981, the Tunisian Dinar was pegged to a currency basket consisting of French Franc, the Deutschmark and the US Dollar. From 1981 to 1986, the Tunisian Central Bank (henceforth, BCT) decided to enlarge the currency basket to achieve greater exchange rate stability. In 1986, the economic recession associated with a deteriorating balance of payment crisis forced the Tunisian authorities to devalue the domestic currency by 10% (Hanna 2001).
From 1986 to 1989, the BCT has lowered the nominal effective exchange rate till the real effective FX rate achieved equilibrium. In the decade of the 90’s, the efforts of authorities were focused on maintaining the stability of the real effective exchange rate. This period was marked by the development of the financial market in Tunisia. Since December 1992, the Tunisian authorities have established the intermediate convertibility of the TND and have launched an interbank FX spot market (March, 1994). During the period 1990-2000, the exchange rate policy has probably contributed to the stability of the Tunisian real effective exchange rate. Indeed, it has successfully contributed to reduce inflation from 8% in 1991 to nearly 3% since 2000 and thus capable to prove the effectiveness of the commitment to macroeconomic stability. Since 2001, because of the Tunisian exchange market’s growing momentum and the decrease in the frequency of the BCT’s interventions (see, IMF report 2006), the latter policy has been shifted to a more flexible one including a managed floating FX rate regime.

Furthermore, the beginning of 2005 was regarded as a first step toward trade liberalization, another interesting reform especially for resident exporting firms who must always repatriate all the cash flows generated by the exports. But, since that date, they were able to maintain 100% of these cash flows in foreign currency accounts. On the other side, the Tunisian banks were also allowed to contract unlimited amount of foreign currency loans. With the aim of providing more derivative products to firms and hedging against FX rate volatility, a forward interbank has been created on July 2007.

Figure 1 shows that the Tunisian interbank market had a relatively stable turnover during the period January 2003 - December 2005. The global turnover amounted to US$ 21.474 million in 2005. The market was largely dominated by spot transactions; they represented 93% of the global turnover. We should note that this part includes both transactions from one foreign currency to another (with a transaction volume of US$ 13.692 million) as well as transactions between foreign currency and the dinar (with a transaction volume of US$ 7.782 million). As shown in Fig. 1, the forward and swap transactions were drastically lower (3% and 2% of the global turnover respectively).

Moreover, interbank’s market share in global turnover was 92% in 2005 (only 85% for 2004). These transactions were conducted by local banks (up to 81%) and by offshore banks (19%). The central bank interventions amounted to US$ 596 million per day in 2005 against US$ 1.151,7 million in 2004. This could be explained by the liquidity appreciation in the interbank market. We should mention that the BCT maintained his policy to closely monitor exchange rate movements during the period (2003-2005) and will directly intervene in
the market in case of any unhealthy fluctuations in the interbank exchange rates due to eventual speculative behavior stemming from any steep decline in the interbank market.

3. Methodological considerations

In this section, we discuss the trend-cycle decomposition procedure and the models that we subsequently use to detect an eventual long memory in the cycle components behavior of Tunisian exchange rates.

3.1. Trend-cycle decomposition

In the empirical research, several techniques have been developed in order to separate a signal into different periodic components. As mentioned above, our approach is inspired by the work of Guy and St Amant (2005). In this sub-section, we provide a brief description of the HP filter (Hodrick and Prescott, 1980), as it is the widely employed in macroeconomics for trend and cycle extraction. For comparison purposes, we also apply the BKfilter (Baxter and King, 1999). It is considered as an ideal pass-band filter, it multiplies the frequencies of interest by 1 and the frequencies outside this band by 0. It is based on concepts of the frequency domain and is applied to the data in the time domain. In our research, we focus our attention on the HP’s methodology as it performs well with our data comparing to the BKfilter. Details regarding BK filter’s results are not reproduced here but are available upon request.

The HP filter is a flexible de-trending method that is widely used in financial and macroeconomics time series models. Formally, let’s assume that the original series $y_t$ is composed of a trend component $(g_t)$ and a cyclical component $(e_t)$. In other words, this filter decomposes the series as follow:

$$y_t = g_t + e_t$$  \(1\)

The Hodrick Prescott’s filter isolates the cyclical component by minimizing the following objective function

$$\sum_{t=1}^{N}(y_t - g_t)^2 + \lambda \sum_{t=2}^{N}((g_t - g_{t-1}) - (g_{t-1} - g_{t-2}))^2$$  \(2\)

Where $\lambda$, smoothness parameter penalizes the variability in the cyclical component. Note that if $\lambda$ is 0, the trend component will be equivalent to the original series while if $\lambda$ diverges to infinity, the trend becomes linear. The choice of the $\lambda$ value depends on the frequency of the data (see Table 1). On the practical side, the commonly used $\lambda$ values are as follows; 1600 for quarterly data and 100 for annual data. Ravan and Uhlig (2002) argue, on the basis of frequency domain considerations, that $\lambda = 1600$ for quarterly data is inconsistent with $\lambda = 100$.
for annual data but would rather correspond to $\lambda = 6.25$. The values of $\lambda$ are obtained by applying this basic formula: $\lambda_x = s^n \lambda_2$.

At present, we have to choose between the following filter methods: (i) HP filter with a 2 power term, (ii) HP filter with a 4 power term as recommended by Uhlig and Ravan (2002), (iii) the BK pass-band filter. In our investigation, we implemented the HP filter with different smoothing parameter $\lambda^1 = 6250000; 6200100; 2250000$ and 1464100 for daily interbank FX rates of EUR, USD, GBP and JPY respectively.

3.2. The long-memory GARCH-type models

The transmission of shocks in an I(0) process occurs at an exponential rate of decay (so that it only captures the short-memory), while for an I(1) process the persistence of shocks is infinite. In the conditional mean the ARFIMA specification has been suggested to fill the gap between short and complete persistence. Subsequently, the short-run behavior of the time series is modeled via the ARMA parameters. However, the fractional differencing parameter (d) allows for modeling the long-run dependence (see, Bollerslev and Mikkelsen 1996, p.158).

Formally, the FIGARCH(p,d,q) model of Baillie et al. (1996) capturing the hyperbolic decay in the volatility process, can be written as follows:

$$
\phi(L)(1 - L)^d \varepsilon_t^2 = \omega + [1 - \beta(L)](\varepsilon_t^2 - \sigma_t^2)
$$

(3)

Where $(1 - L)^d$ is the fractional differencing operator, defined as follows:

$$
(1 - L)^d = \sum_{k=0}^{\infty} \frac{r(d+1)L^k}{r(1)^r(d-k+1)} \\
= (1 - dL) - \frac{1}{2} d(1 - d)L^2 - \frac{1}{6} d(1 - d)(2 - d)L^3 - \cdots \\
= 1 - \sum_{k=1}^{\infty} c_k(d)L^k
$$

(4)

where $c_1d = d$, $c_2d = \frac{1}{2} d(1 - d), \ldots$ etc.

In our investigation, we employed the hyperbolic GARCH (HYGARCH), which was introduced by Davidson (2004) as a generalization of the fractionally integrated GARCH model (FIGARCH) with hyperbolic convergence rates. With reference to Davidson (2004), the FIGARCH model does not specify a covariance

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1Applying Ravan and Uhlig (1997)’s technique, the daily parameter is equal to the exact number of day per year exponent 2 and multiplied by 100 (annual smoothing parameter $\lambda$). We have 250 days for the TND/EUR, 249 days for the TND/USD, 121 days for TND/GBP and 121 days for the TND/JPY.
stationary process when \( d > 0 \). However, HYGARCH process allows the existence of second moments at more extreme amplitudes than the integrated GARCH (IGARCH) and FIGARCH models. More specifically, when \( k = 1 \) the HYGARCH model nests the FIGARCH, when \( 0 < k < 1 \) this process is stationary, and when \( k > 1 \) this process is non-stationary.

In order to derive the HYGARCH model, Davidson (2004), considers the following form;

\[
\theta(L) = 1 - \frac{\delta(L)}{\beta(L)} \left( 1 + \alpha((1 - L)^d - 1) \right)
\]

\( \alpha \geq 0, d \geq 0 \) \hspace{1cm} (5)

Note that when \( d > 0 \),

\[
S = 1 - \frac{\delta(1)}{\beta(1)} (1 - \alpha)
\]

The FIGARCH and stable GARCH cases correspond to \( \alpha = 1 \) and \( \alpha = 0 \) respectively. The hypothesis of either of these two cases might be tested. However, in the latter case the parameter \( d \) is unidentified, which poses a well known problem for constructing hypothesis tests. Therefore, when \( d = 1 \), Eq. 5 will be reduced to the following form;

\[
\theta(L) = 1 - \frac{\delta(L)}{\beta(L)} (1 - \alpha L)
\]

\( \alpha \geq 0 \) \hspace{1cm} (7)

In other words, when \( d = 1 \), the parameter \( \alpha \) reduces to an autoregressive root, and the model will correspond to whether a GARCH or stable IGARCH process, depending on \( \alpha \) value (\( \alpha < 1 \) or \( \alpha = 1 \) respectively).

When \( d \) is not too large, this model will correspond closely to the case;

\[
\theta(L) = 1 - \frac{\delta(L)}{\beta(L)} (1 - \alpha \varphi(L))
\]

\( \varphi(L) = \zeta(1 + d)^{-1} \sum_{j=1}^{\infty} j^{-1-d} L^j \hspace{1cm} d > 0 \) \hspace{1cm} (8)

where \( \zeta(.) \) is the Riemann zeta function.

4. Empirical findings

Before testing for long memory in the daily returns and volatility series, we decompose the actual FX rate time series based on the two selected filters HP and BK. Sub-section 4.1 presents the data and some descriptive statistics while the empirical results relative to the decomposition of the original FX rates are displayed and discussed in sub-section 4.2.

4.1. Data and preliminary analysis
The augmented Dickey-Fuller test (ADF), (Dickey and Fuller 1979) and the corresponding Philips-Perron test (PP), (Philips and Perron, 1988) are used to test for a unit root in the individual interbank FX rates. The results indicate that we could not reject the unit-root hypothesis at the 5% level (the full test results are available upon request).

Here, we should note that the first innovative side of our study is the use of a new procedure to investigate the trend and the original series extraction. Indeed, it is important to bring up a feature that occurs frequently in financial time series: The non stationary of empirical data especially data on the FX rates. Therefore, many researchers focused on the identification and the extraction of the trend to portray this non stationarity which have led to bringing out a popular method of decomposition trend-cycle type: stationary differenced series (linear trend) which is used by most of the major studies and stationary output gap series (stochastic trend) which is involved in the present study. Applications of trend-cycle decompositions to FX rates are discussed below.

Some descriptive statistics are used to analyze the basic features of the cycle component series gathered from our study. The original daily series, HP2 trend with the respective smoothing parameters and their cyclical components are plotted in Fig. 3. For space scarcity and readability considerations, we provide only the case of the TND/USD rate. The other figures are available upon request. The output gap is the percentage difference between actual output and the potential output. According to Conway and Frame (2000), “Potential output represents an economy’s steady-state level of output that is the level of output to which actual output reverts in the absence of temporary shocks…In the framework of monetary policy; the output gap provides a measure of inflationary pressure in the economy” (Conway and Frame 2000, p. 2).

With reference to Guy and St Amant’s (2005) framework, the HP2 is the preferred filter. The output gap estimate displays the fact that the Tunisian economy has been subject to severe economic shocks that create unstable environment over vast periods of time. From this point, we can observe that the output gap estimates is marked by very sharp cyclical turning points. They are approximately symmetrical around the zero line. The upper panel in these Figs. also shows that the oscillations trends are too small and exhibit peaks and declines with very low amplitude (their level are very close to zero) over the full sample period. The percentage deviations of the original series TND/EUR from the HP trend are not large. They rather approach the zero line. We highlight that this parity is more stable than the others (its deviation value is 0.01397). TND/JPY parity appears more volatile than the others parities (its deviation value is 0.03).

Table 3 displays some descriptive statistics for the cyclical component series. The last two lines report the Jarque-Bera normality statistic (Jarque and Bera,1979) and the associated p-value. This test a “goodness-of-fit”
measure of departure from normality based on the sample kurtosis and skewness. The negative skewness statistics indicate the possibility that the considered series has a negatively skewed distribution and the positive values indicate the possibility that the series has a positively skewed distribution. Moreover, all the cyclical component time series are characterized by excess of kurtosis.

We perform unit root tests for the cyclical component series in order to specify univariate properties of the series. The test results are provided in Table 4. From these results, we conclude that both ADF as well as KPSS (Kwiatkowski et al., 1992) tests confirm that the time series are level stationary.

4.2. Decomposition of FX rate time series

Each FX rate time series will be decomposed into permanent and transitory components using three detrending methods, namely HP2, HP4 and BK filters. Hence, we assume that business cycle corresponds to variations in which the period was comprised between the limits \( P_L \) and \( P_U \), where \( P \) denotes the length of the cycle while \( P_U \) and \( P_L \) represent respectively the upper and the lower limits. We should indicate that we have selected a specific interval to each series.

Beforehand, we observe that the obtained \( P_U \) values are very large (expressed in day’s unit). By convention, low frequency values will approximate zero. We then compute the output gap (transitory component) between the line representing the original series and its HP or BK trend (permanent component). Plots for cyclical component series are not shown here in the interests of brevity (as mentioned before, only the case of TND/USD cyclical component extracted by HP2 filter is shown in Fig. 3). Having a look on the graph plotting all the cyclical components, we observe that the general shape of the cycles extracted by Baxter and King’s filter are very similar to the ones extracted by HP4 filter for TND/GBP and TND/JPY exchange rates. Gaps extracted by HP2 and HP4 filters remain in phase with each other. However, this later exhibits daily cycle with larger amplitude. These two de-trending methods lead to highly correlated cyclical amplitudes for TND/EUR and TND/JPY respectively but a middle correlation for the remaining exchange rates is detected.

The extraction efficiency of these filters has been evaluated using spectral analysis which is the most frequently used method for this purpose. For that reason, Guy and St Amant (2005) focus on the evaluation of HP and BK filters’ efficiency for low frequency macroeconomic time series. They show that it is reasonable to verify two restrictions in an attempt to isolate any cyclical component of time series; they suppose firstly that if we have low frequency data and apart from the fact that the spectrum of these series exhibit a peak located at the

\[ \text{Most of macroeconomic time series have a typical Granger shape, where it is well-known that most of the power occurs at very low frequencies.} \]
range of the business cycle frequency, filters do not produce satisfying results. Secondly, the authors impose another restriction related to the spectrum of the output gap. Indeed, the spectrum of the gap series should be a non-decreasing function of frequency (the power spectral density increases proportionally to the frequency) in the range of business cycle frequency.

The spectrogram plot of each of the four exchange rate time series\(^3\) clearly show the existence of a main peak (at this point the power spectrum reaches its maximum value) located at zero frequency\(^4\) belonging to the business-cycle frequency bands. This preliminary result is in line with the one provided by the authors. However, in their case study, the peak is located at zero frequency which is outside the quarterly business cycle frequency band (0.05-0.35). It is obvious that the first condition imposed by Guy and St Amant (2005) is satisfied and the application of the filters provides reliable results.

The estimated spectrums of each exchange rate cyclical component isolated by each filter are plotted in Fig. 2 (as we get similar results for all the series, we only report the TND/USD spectrums’ figures). The spectrums of HP4 and BK de-trended series are analogous with the original data spectrum (They possess low frequency resonance peak and they are a decreasing functions)\(^5\). This was accompanied by a progressive decrease in the power of the output gap spectral density, especially in the business cycle frequency band. However, we notice that the cyclical components extracted by HP2 filter are non-decreasing functions (since the spectral density increases only and especially in the business cycle frequency band). Therefore, we may conclude that only HP2 filter fulfills the second condition imposed by the authors for all the series.

The area positioned underneath each curve spectrum represents the variance in the original data (King and Watson, 1996). The peaks produced by the spectrums are, in general, located in the business cycle frequency range and they correspond to the frequencies that contribute the most to the original data variations\(^6\). On the contrary, peaks outside the business cycle frequency range can be attributed to the presence of non cyclical frequencies that account for the most important contribution to the variance. Three possible explanations of such peak could exist; the non stationarity, the transitory phenomena and the stationarity with long run dependence. For the other filters, spectrums do not show peaks inside the business cycle frequency band. It is clear then that

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\(^3\) These figures are also not reported.

\(^4\) All the time series have roughly the same spectrum form as Granger’s typical spectral shape. They are dominated by low frequencies since we use daily data.

\(^5\) With reference to the results provided by Guy and St Amant (2005), cyclical component spectrum present peaks located at low frequencies. Then their shape is not similar to the one of non filtered series.

\(^6\) This definition was suggested as a standard conception by various authors (as for example Levy and Dezhbakhsh, 2003, p. 418), who said, in a previous paper dealing with a spectral analysis of economic variable (annual frequencies) “a spectral peak in the business cycle frequency range \(0.785 \leq \omega \leq 2.06\) suggest that business cycles contribute to much of the series variations.”
the extracted cyclical components exhibit series with non cyclical main deviations, whereas, the HP2 output gap series is cyclical. HP2 filter performs well in terms of identifying the cyclical component of exchange rate series.

4.3. Long memory analysis

4.3.1.1. Some preliminary results

The preliminary test consists of investigating whether the obtained cyclical component series exhibit long memory distribution by plotting their autocorrelograms (not reported) after taking their absolute values. According to Taylor (1986), the volatility as measured by absolute variations shows a strong autocorrelation than that as measured by the squared variations. The results show that there is no evidence of long-range dependence in the series themselves, but in their absolute values. A flexible and more parsimonious way to capture the behavior of these series is by means of an ARFIMA \((p,d,q)\) model. Before we calibrate the model, we need to check whether or not it should still be possible to maintain our basic assumption for the long memory in the volatility of the cyclical component series. To set sights on this, let us perform possible long memory tests.

4.3.1.2. Tests for \(I(0)\) against \(I(d)\): 

From a graphical demonstration, long memory phenomena are a priori present in the volatility of the cyclical component. To further verify this hypothesis, we should use some conventional long memory tests. For this reason, we apply a new version of test on these series, namely HML which is established by Harris et al. (2008). These authors demonstrate that the HML statistic is asymptotically normal \(N(0,1)\) under the null hypothesis of short memory. According to the authors, this statistic “is based on only high-order sample autocovariances and and by construction eliminates the effects of nuisance parameters typically induced by short memory autocorrelation”. Two parameters need to be selected with caution to achieve adequate fit; a truncation parameter \(c\) where \(k=(cT)^{1/2}\) is the lowest order lag, and \(L\) where \(l=LT^{12/25}\) is the bandwidth truncation of variance parameter.

For comparison purposes, two other well-known tests for long memory are included in the analysis; the modified R/S statistic of Lo (1991)\(^7\) and the KPSS test of Kwiatkowski et al. (1992), extended by Lee and Schmidt (1996). Each of these two statistics tests the unit root null hypothesis against the long memory alternative. As it is shown by Lee and Schmidt (1996) both tests are sensitive to the choice of lag truncation i.e. the number of covariance terms used to calculate the long run variance. Table 5 shows the results of Lo, KPSS

\(^7\) Lo (1991) demonstrates that short-term dependence can produce biased results in traditional R/S test proposed by Hurst (1951) and therefore suggested the modified R/S as an alternative one.
and HML tests conducted on absolute cyclical component series. Based on the asymptotic critical values of the modified R/S and KPSS, it is well documented that the filtered series in their absolute values exhibit long memory behavior. Similar conclusion can be derived from the estimated HML statistics. Indeed, to check the sensitivity of our results to the choice of the truncation \( n \), we report statistics based on \( T^{0.5} \), \( T^{0.55} \) and \( T^{0.6} \).

The results indicate that the evidence of long memory in all the series is qualitatively the same across different choices of \( n \), since all the calculated statistics are more extreme than the standard normal critical value of 1.96 for the appropriate values of “c” and “l”. In sum, we find evidence of long memory in the output gap of individual exchange rate series. These findings warrant the use of ARFIMA models to examine the dynamics of these series.

4.3.2. Estimation of ARFIMA process

In order to capture long memory features of the series, we consider one of the most popular long memory process; ARFIMA\((p,d,q)\) as suggested by Granger and Joyeux (1981). Two widely used estimation procedures for the ARFIMA process are; the one-step method namely maximum likelihood technique and the two-step method of spectral regression (GPH) which is proposed by Geweke and Porter-Hudak (1983). Within the class of parametric regression procedures, the most used technique to estimate ARFIMA process is the maximum likelihood method (ML). Despite the fact that it is theoretically more straightforward, in practice it can be quite difficult to achieve due to the complexities of calculating the inverse of the variance-covariance matrix (see, Sowell, 1992a).

To estimate ARFIMA model, we applied the Sowell’s three-step procedure (see, Sowell, 1992a, p.175) in order to identify the initial parameters values \( p, d \) and \( q \). In the first step, we estimate the fractional differencing parameter \( d \) using different estimators such as GPH and Whittle. In the second step, we determine the appropriate roots of the given the autoregressive (AR) and moving average (MA)- polynomials for each truncated series \((1 – L)^d \times Y_t\). We choose a range of values for the estimated fractional differencing parameter \( d \). We should mention that for every initial value of \( d \), we compute the truncated series \((1 – L)^d \times Y_t\) and we estimate ARMA parameters, \( d \) and the white noise variance. Finally, the parameters, in each ARFIMA\((p,d,q)\) model, are estimated by ML for which the log likelihood achieve its maximum value. Once the convergence is ensured, we select the appropriate ARFIMA specifications using the Akaike (AIC, Akaike, 1974) and Schwartz-Bayesian (SIC, Schwartz, 1978) information criteria.

Following Sowell (1992), we first estimate the long memory parameter \( d \) separately from the AR and MA parameters using GPH. It should be noted, however that according to Agiakloglou et al. (1993),the GPH
estimator may suffer from a number of very severe deficiencies. Thus, to check the robustness of our estimations, we employed the bias test suggested by Davidson and Sibbertsen (2006). These authors proved that under the null hypothesis of no bias, the statistics have a standard normal distribution. It is then interesting to compare the values obtained by the GPH regressions and those obtained by other different regression methods. For that purpose, we estimate the long memory parameter using Whittle (1951) and Moulines and Soulier (1999) (MS)’ regressions. Empirical results of the long memory parameters estimated by GPH (with the associated bias tests), MS and Whittle are available upon request. In general, we find that the GPH estimator is appropriate for stationary long memory process with \(-0.5 < d < 0.5\). Each series is subject to the transformation \((1 - L)^d\) where long memory parameters “\(d\)” are equal to 0.5, 0.55 and 0.6 respectively.

At this stage, following the standard Box and Jenkins (1970)’s procedure, it is usual to choose initial values of AR and MA parameters for each model. A careful inspection of both ACF and PACF shapes (these figures are not reported here) reveal that all the truncated series seem following a suitable AR process-type of behavior. Regarding these figures, the choice of the AR terms \(p\), is made as follows; 1, 2 and 3 for the TND/EUR FX rate, 1 and 2 for the USD and only one lag order for GBP and JPY respectively. Finally, we estimate the different ARFIMA models in terms of the obtained AR lags (assuming that the innovations are non-normally distributed) and we would then choose the optimal ARFIMA(\(p^*, d^*, q^*\)) orders that minimizes the information criteria among all possible subsets. Parameter estimates obtained from the ML method are displayed in Table 6. Using the AIC and log-likelihood criteria, the optimal ARFIMA parameterizations are: ARFIMA(3,0.5,0), ARFIMA(2,0.5,0), ARFIMA(1,0.55,0) and ARFIMA(1,0.6,0) models relatives to TND/EUR, TND/USD, TND/GBP and TND/JPY respectively.

These results confirm that there is strong evidence of long memory in the cyclical components of the FX rates (except the Euro). More precisely, the estimated fractional integrated parameters \(d\) are found to be positive and significant at 1% level. However, for the euro parity, the results show that the long memory parameter is statistically significant, but negative. This implies that long memory in the conditional mean is not a robust feature for the EUR’s cyclical components which is not in accordance with the GPH test results. Similarly, we indicate that all the AR terms are highly statistically significant. Moreover, the LM test is implemented for the standardized residuals in order to detect an eventual presence of ARCH effects. Obviously, this test fails to accept the null hypothesis of no serial heteroskedasticity in residuals (except for the GBP parity). Since our objective is to assess the relevance of dual long memory in the series, we re-estimate ARFIMA models simultaneously with four models for the conditional variances.
4.3.3. Estimation results of dual long memory models

We place a particular emphasis in this paper on estimating the dual long memory property of the filtered series and their volatilities using the ARFIMA model for the conditional mean jointly with different long memory GARCH-type models for the conditional variance, namely FIGARCH and HYGARCH.

The estimation results of all the joint models for the selected interbank FX rate time series are displayed in Table 7. Several findings emerge from these results. Based upon the AIC, SIC and HQC criteria, we note that the selected ARFIMA(1,dm,0) and ARFIMA(2,dm,0) estimated jointly with the HYGARCH(1,dv,1) outperform the other models for the JPY and for the USD respectively. While for the EUR and GBP, the same information criteria come out in favor of the simultaneous estimation of FIGARCH(0,dv,0) models respectively with AR(3) and ARFIMA(1,dm,0)\textsuperscript{9}.

The evidence for the remaining three cyclical components significantly exhibits the long memory property. It is obvious from Table 7 that for all those three currencies (GBP, JPY and USD) the long memory parameters dm in the mean equations, are significantly varying between 0.22 and 0.3 smaller than 0.5 implying a stationary process. Moreover, the estimated parameters dv in the conditional variances are smaller than 0.5 and ranging between 0.2 and 0.4. Consequently, the conditional variance dynamics of these time series are also governed by a stationary process.

In addition, the obtained results pointed out that the estimated ARCH/GARCH coefficients are statistically highly significant and their sums are close to one. Moreover, the Student parameters $\nu^{1/2}$ are statistically significant for all the models indicating the presence of fat tails. Also, the hyperbolic terms $\alpha$ in the HYGARCH models are statistically significant and they satisfy the stationarity restriction (see Davidson, 2004). According to Box-Pierce (1970) statistics, the null hypothesis of uncorrelated standardized and squared standardized residuals cannot be rejected for all the selected models.

Overall, our results suggest that dual long memory properties are important characteristics of three out of the total four output gaps of the exchange rates. In other words, the paper shows that monetary shocks captured by the transitory components are largely permanent.

\textsuperscript{8} The estimation results of the remaining possible models are available upon request.

\textsuperscript{9} The estimation of the first selected specification for the EUR (ARFIMA(3,dm,0)-FIGARCH(0,0)) reveal insignificant AR parameters. Then, the model is rejected, although $d_m$ (the estimated long memory parameters for the conditional mean), and $d_v$ (for the conditional variance), are significant. Consequently, the AR(3)-FIGARCH (0,0) specification can be conveniently applied for the EUR.
5. Summary, Discussion and major implications

Fluctuations in nominal FX rates have traditionally been viewed as transitory deviations from a deterministic time trend. At this stage, empirical research must continue to keep alive the yet unsolved debate on whether real and temporary components exhibit a long or short run memory structure. This would raise important questions about how the memory structure is for the transitory component. This research issue seems to be widely neglected in the literature especially for emerging markets. The aim of this study is to focus on the transitory component and to specify whether this shock is long-lasting or no. Our study is focused on the Tunisian interbank FX market.

On the practical side, the cyclical component volatilities of Tunisian FX rates are estimated using various long memory GARCH-type models. The research is especially interested in the predictive performance of HYGARCH model. In this respect, we first implement the Guy and St Amant (2005)’s procedure in order to extract the transitory component series. By examining their spectral shapes, our study reveals that HP filter with two power terms performs better in spelling up these time series. Then, we have explored the nature of these series by examining their memory properties. We check the robustness of our findings in a twofold manner. First, we apply the recent HML test proposed by Harris et al. (2008) to test for long memory. Second, we take the advantage of the bias test introduced by Davidson and Sibbertsen (2006) to show the robustness of the GPH estimator. We finally examine long memory properties in both conditional mean and conditional variance of the filtered series.

The presence of long memory in the cyclical components entails that monetary shocks to Tunisian FX rates do not have a short-run transitory effects but rather long term effects. This finding should be considered with caution. Indeed, many empirical studies maintain two fundamental premises: real shocks have a permanent effect on exchange rates, whereas monetary (nominal) shocks have only temporal effect. Consequently, monetary policy may be a more effective tool since it can react faster to changes in macroeconomic conditions. According to our results and without neglecting the role of real shocks often justified in literature (Daly (2006) among others), one can also attribute a major role to monetary shock which could also have permanent effect on the FX rate. Therefore, in order to stabilize prices, policy makers should react to monetary shocks in the same way as real shocks.
In other words, the persistence of cyclical exchange rates’ deviations can be explained by the presence of anomalies (imperfect information) resulting in an inability to perfectly disentangle between the permanent and transitory shocks. More explicitly, the signal may convey irrelevant or meaningless information while relevant and useful information may have been omitted (Andersen and Biere, 2005). Hence, the market inefficiency appears able to hinder the Tunisian foreign exchange market.

If the efficient market hypothesis holds, current exchange rates would rapidly adjust to presently unknowable information and they would move randomly in the future. Lee and Lee (2009) pointed out that if prices follow a mean reverting process, then there is a tendency for the price level to return to its trend path over time and that it may be possible to predict future price movements based on past behavior. By contrast, if prices follow a random walk process, then any shock to prices is permanent and future rates cannot be predicted by analyzing historical movements.

Some authors (such as Caporale et al. 2009) focus mainly on the claims that trade liberalization and abolition of exchange controls in many developing countries have a destabilizing effect on real exchange rates while others (such as Aguirre and Calderon 2005; 2006) declare that the degree of openness can play an extremely useful role in stabilizing exchange rates fluctuations. Regarding our results, the long lasting cyclical shocks (reflecting instability) identified in the Tunisian FX rates testify that “financial liberalization should be pursued only gradually” (Caporale et al. 2009; page 3) and new strategic plans must be implemented in order to meet the challenge posed by financial integration. More specifically, country should be better positioned to attract more foreign investments and hence encouraging domestic ones. Liberalization can also improve the market liquidity and generate more effective risk diversification.

On the other hand, in the presence of imperfect information, there could exist profitable arbitrage opportunities between Tunisian exchange rates. For example, consistent with the view that domestic investors have information advantages over foreign investors, then when relevant new information becomes available, foreign investors tend to raise capital in their domestic markets more. Thus, domestic investors have naturally superior information about the fluctuation of exchange rates (Lee and Lee 2009). However, due to the lack of information about the dynamics of exchange rates, investors seeking opportunities in developing countries will be much more vulnerable to risk (Alagidede and Panagiotidos, 2009). Each foreign investor will be disinterested in investing in Tunisia which raises concern about a possible decrease in Tunisian foreign exchange market liquidity.
Another interesting feature of this study is that the data period (from 1997 to 2005) is chosen to cover the Asian currency crisis of 1997-98. Importantly, Caporale et al. (2009) pointed out that for developing countries including Tunisia, external shocks are among the important factors influencing exchange rate movements. Lim et al. (2008) investigates the impact of the Asian financial crisis on the efficiency of eight Asian stock markets. Their results indicate that the crisis adversely affect the efficiency of most Asian stock markets. According to them, under the assumption that non linear serial dependencies indicate equilibrium deviation resulted from external shocks, the evidence for higher inefficiency detected during the crisis are likely to result from the chaotic financial environment at that time and investors would overreact not only to local news, but also to news originating in the other markets. Our results suggest that the Asian crisis may have contagious impact on the Tunisian currency. Indeed, as we demonstrated in section 4.1, the JPY/TND exchange rate exhibits the highest standard deviation whereas the EUR/TND exchange rate appears to be the most stable.

Our findings present some major financial and policy implications for FX markets in small emerging countries. First of all, detecting long memory in foreign exchange rates’ cyclical components could provide another promising approach to forecast FX rates in emerging countries (Harris Stoja and Yilmaz 2011). Moreover, extracting these trends from FX rates offers useful information to explain the future variations of interbank FX rates. In small FX markets, underlying factors such as inflation, interest rates, trade balance account, budget deficit, competitiveness, that drive these trends are seriously considered by central bank policies.

Secondly, from a market microstructure view, the presence of long memory in the transitory components of FX rates may provide important insights into the relationship between interbank turnover and daily FX volatility. In this perspective, trading volume may be included as a plausible variable to explain the obtained output gaps.

Finally, in a modeling perspective, exploring the presence of long memory in FX rates’ cyclical components can play a key role in helping to achieve superior forecasting performance than other standard models. More precisely, we may explain the well-known inability of structural models to consistently capture long memory in FX rates in a linear way by their failure to outperform predictions of a random walk process.

References


Guy, A., St-Amant, P., 2005. Do the Hodrick-Prescott and Baxter-King Filters provide a Good Approximation of Business Cycles, the Annales d'économie et de statistique 77: 133-155, ISSN 0769-489X.


**Appendix**

**Table 1**

<table>
<thead>
<tr>
<th>Frequencies</th>
<th>$\lambda(n=2)$</th>
<th>$\lambda(n=4)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yearly</td>
<td>100</td>
<td>6.25</td>
</tr>
<tr>
<td>Quarterly</td>
<td>1600</td>
<td>1600</td>
</tr>
<tr>
<td>Monthly</td>
<td>14400</td>
<td>129600</td>
</tr>
</tbody>
</table>

**Table 2**

Sample periods of the Tunisian interbank FX rates (daily frequency)

<table>
<thead>
<tr>
<th>Interbank FX rate</th>
<th>Sample range</th>
<th>No. of Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>EUR/TND</td>
<td>From Jan 1999 to Dec 2005</td>
<td>1747</td>
</tr>
<tr>
<td>USD/TND</td>
<td>From Jan 1997 to Dec 2005</td>
<td>2241</td>
</tr>
<tr>
<td>GBP/TND</td>
<td>From Jan 1997 to Dec 2005</td>
<td>1355</td>
</tr>
<tr>
<td>JPY/TND</td>
<td>From Jan 1997 to Dec 2005</td>
<td>1086</td>
</tr>
</tbody>
</table>

_TND is the Tunisian Dinar_

**Table 3**

Descriptive statistics for cyclical component of the Tunisian interbank FX rates

<table>
<thead>
<tr>
<th>TND/EUR</th>
<th>TND/USD</th>
<th>TND/GBP</th>
<th>TND/JPY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-4.71e-11</td>
<td>-7.24e-11</td>
<td>-2.87e-12</td>
</tr>
</tbody>
</table>
### Table 4
Unit-root and stationarity test results for the cyclical component of the Tunisian interbank FX return time series

<table>
<thead>
<tr>
<th>Tunisian interbank FX rates</th>
<th>ADF test</th>
<th>KPSS test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varphi_\mu$</td>
<td>$\varphi_\zeta$</td>
</tr>
<tr>
<td>EUR/TND</td>
<td>-6.85***</td>
<td>-6.85***</td>
</tr>
<tr>
<td>USD/TND</td>
<td>-6.12***</td>
<td>-6.12***</td>
</tr>
<tr>
<td>GBP/TND</td>
<td>-5.77***</td>
<td>-5.77***</td>
</tr>
<tr>
<td>JPY/TND</td>
<td>-5.59***</td>
<td>-5.86***</td>
</tr>
</tbody>
</table>

Notes: The $\varphi_\mu$ and $\varphi_\zeta$ statistics refer to the ADF and KPSS tests respectively. The subscripts $\mu$ and $\zeta$ indicate the models that allow for a drift term and both a drift and deterministic trend respectively for a lag length of 4. (*), (**) and (*** are respectively the significance at 10%, 5% and 1% level. The critical values are extracted from MacKinnon (1991) for the ADF test and from Kwiatkowski et al. (1992) for the KPSS test. ADF test examines the null hypothesis of a unit root against the stationary alternative. KPSS tests the stationarity null hypothesis against the alternative hypothesis of a unit root.

### Table 5
The long memory test results

<table>
<thead>
<tr>
<th>The interbank FX rates</th>
<th>Lo’s test</th>
<th>KPSS test $T^{0.5}$</th>
<th>KPSS test $T^{0.55}$</th>
<th>KPSS test $T^{0.6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TND/EUR</td>
<td>3.6718</td>
<td>2.364</td>
<td>2.0552</td>
<td>2.0196</td>
</tr>
<tr>
<td>TND/USD</td>
<td>3.3474</td>
<td>1.1958</td>
<td>2.1567</td>
<td>2.1411</td>
</tr>
<tr>
<td>TND/GBP</td>
<td>2.6456</td>
<td>2.0167</td>
<td>2.1437</td>
<td>2.0862</td>
</tr>
<tr>
<td>TND/JPY</td>
<td>2.8416</td>
<td>0.6229</td>
<td>2.0784</td>
<td>2.0392</td>
</tr>
</tbody>
</table>

Notes: The R/S and KPSS critical values at 5% significance level are 1.747 and 0.463 respectively. For HML test, bandwidth c value is 0.25 for the parities TND/JPY and TND/GBP respectively, 0.15 for the parities TND/EUR and 0.1 for the TND/USD.

### Table 6
ARFIMA model estimating results (Maximum likelihood method)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>TND/EUR ($T^{0.5}$)</th>
<th>TND/USD ($T^{0.5}$)</th>
<th>TND/GBP ($T^{0.5}$)</th>
<th>TND/JPY ($T^{0.5}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>-0.23***</td>
<td>-0.30***</td>
<td>-0.12***</td>
<td>-0.23***</td>
</tr>
<tr>
<td></td>
<td>(-4.09)</td>
<td>(4.38)</td>
<td>(2.73)</td>
<td>(4.84)</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>0.99***</td>
<td>0.99***</td>
<td>0.98***</td>
<td>0.82***</td>
</tr>
<tr>
<td></td>
<td>(2280)</td>
<td>(358)</td>
<td>(434)</td>
<td>(19.7)</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.74***</td>
<td>1.07***</td>
<td>0.88***</td>
<td>0.85***</td>
</tr>
<tr>
<td></td>
<td>(12.1)</td>
<td>(14.0)</td>
<td>(32.5)</td>
<td>(28.2)</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.12***</td>
<td>-0.15***</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(3.71)</td>
<td>(-2.72)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>0.08***</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(2.38)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes: J–B is the Jarque–Bera test statistic for testing whether the series is normally distributed.
Table 7
The maximum likelihood estimating results of dual long memory models using the BFGS algorithm

<table>
<thead>
<tr>
<th>Parameters</th>
<th>TND/EUR</th>
<th>TND/USD</th>
<th>TND/GBP</th>
<th>TND/JPY</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu)</td>
<td>0.996***</td>
<td>0.995***</td>
<td>0.990***</td>
<td>0.872***</td>
</tr>
<tr>
<td>(1993)</td>
<td>(454.5 )</td>
<td>(252.7 )</td>
<td>(20.11)</td>
<td></td>
</tr>
<tr>
<td>(d_0)</td>
<td>-</td>
<td>0.309***</td>
<td>0.270***</td>
<td>0.224***</td>
</tr>
<tr>
<td>(9.12)</td>
<td>(4.38)</td>
<td>(4.65)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\phi_1)</td>
<td>1.233***</td>
<td>1.073***</td>
<td>0.853***</td>
<td>0.889***</td>
</tr>
<tr>
<td>(51.92)</td>
<td>(26.19)</td>
<td>(20.3 )</td>
<td>(33.23)</td>
<td></td>
</tr>
<tr>
<td>(\phi_2)</td>
<td>-0.215***</td>
<td>-0.158***</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(-6.56)</td>
<td>(-4.003)</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>(\phi_3)</td>
<td>-0.047***</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(-2.28)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>(\omega)</td>
<td>1.082***</td>
<td>15.88***</td>
<td>1.774***</td>
<td>0.017***</td>
</tr>
<tr>
<td>(10.03)</td>
<td>(32.3 )</td>
<td>(15.94)</td>
<td>(5.75)</td>
<td></td>
</tr>
<tr>
<td>(\hat{\nu})</td>
<td>0.094***</td>
<td>0.400***</td>
<td>0.265***</td>
<td>0.340***</td>
</tr>
<tr>
<td>(5.60)</td>
<td>(2.04)</td>
<td>(4.42)</td>
<td>(1.72)</td>
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</tr>
<tr>
<td>(\alpha)</td>
<td>-</td>
<td>1.440***</td>
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<td>1.023***</td>
</tr>
<tr>
<td>(3.96)</td>
<td>-</td>
<td>(7.68)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\delta_1)</td>
<td>-</td>
<td>0.293***</td>
<td>-</td>
<td>0.370***</td>
</tr>
<tr>
<td>(2.53)</td>
<td>-</td>
<td>(1.782 )</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>(\beta)</td>
<td>-</td>
<td>0.829***</td>
<td>-</td>
<td>0.613</td>
</tr>
<tr>
<td>(11.51)</td>
<td>-</td>
<td>(2.229 )</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>(\nu^{1/2})</td>
<td>2.330***</td>
<td>3.020***</td>
<td>2.041***</td>
<td>2.296***</td>
</tr>
<tr>
<td>(8.20)</td>
<td>(10.50)</td>
<td>(16.36)</td>
<td>(12.56)</td>
<td></td>
</tr>
<tr>
<td>SIC</td>
<td>10321.1</td>
<td>12384.3</td>
<td>5671.56</td>
<td>1983.37</td>
</tr>
<tr>
<td>AIC</td>
<td>10340.2</td>
<td>12412.9</td>
<td>5687.16</td>
<td>2005.77</td>
</tr>
<tr>
<td>HQC</td>
<td>10333.1</td>
<td>12402.4</td>
<td>5681.32</td>
<td>1997.29</td>
</tr>
<tr>
<td>Log-Likelihood</td>
<td>10347.2</td>
<td>12422.9</td>
<td>5693.16</td>
<td>2014.77</td>
</tr>
<tr>
<td>Q(25)</td>
<td>5.192</td>
<td>37.1065</td>
<td>26.6201</td>
<td>23.4479</td>
</tr>
<tr>
<td>Q^2(25)</td>
<td>0.0203</td>
<td>25.7864</td>
<td>7.5632</td>
<td>39.1623</td>
</tr>
</tbody>
</table>

Notes: Values in parentheses indicate t-statistics. *, ** and *** denote statistical significance at 10, 5 and 1% respectively. Q(25) and Q^2(25) are the Box-Pierce test statistics for the first 25 autocorrelations applied to the residuals and squared residuals respectively. HQC indicates Hannan-Quinn criteria developed by Hannan and Quinn (1979). BFGS is the Broyden-Fletcher-Goldfarb-Shanno Algorithm.
Figure 2
Spectral Shape of exchange rate cyclical component series (HP2, HP4 and BK) (Spectral density as a function of frequency in Hertz)

<table>
<thead>
<tr>
<th>HP2</th>
<th>HP4</th>
<th>BK</th>
</tr>
</thead>
</table>

FX rate: TND/USD (business cycle range: 0-0.002)

Figure 3 - TND/USD, HP2 trend and output gap series