On the dynamic dependence between US and other developed stock markets: An extreme-value time-varying copula approach

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February 10, 2014

Abstract: This paper examines the dynamic dependence between American and four developed stock markets, namely, Japan, United Kingdom, Germany and France during a recent period including the global financial crisis 2007-2009. The econometric approach is based on the extreme-value time-varying copula functions. Specifically, the marginal distributions are reproduced by an extreme-value based model while the joint distribution is explored using time-varying Normal and SJC copulas. The empirical results show that the dynamic dependence between American and Japanese stock markets is symmetric while that between American and European stock markets is asymmetric. In particular, this dependence seems to be related to geographic position.

Keywords: dependence, stock markets, extreme value theory, time-varying copulas.

Résumé: Ce papier examine la dépendance dynamique entre le marché des actions américain et ceux de quatre pays développés, à savoir, le Japon, le Royaume-Uni, l’Allemagne et la France au cours d’une période récente qui inclut la crise financière mondiale de 2007-2009. L’approche économétrique repose sur les fonctions copules basées sur les valeurs extrêmes et variables dans le temps. Plus précisément, les distributions marginales sont reproduites par un modèle basé sur les valeurs extrêmes alors que la distribution conjointe est explorée à l’aide des copules normales et SJC variables dans le temps. Les résultats empiriques montrent que la dépendance dynamique entre le marché des actions américain et japonais est symétrique alors que celle entre le marchés des actions américain et européen est asymétrique. En particulier, cette dépendance semble être liée à la position géographique.

Mots-clés: dépendance dynamique, le marché des actions, la théorie des valeurs extrêmes, copules variables dans le temps.
1 Introduction

The dependence structure between stock markets has been largely investigated in the literature because of its important implications for several financial applications such as international diversification, market integration, contagion effect, risk management, asset pricing, and portfolio allocation.

Numerous empirical studies have been developed and various econometric approaches have been considered to explore the dependence between American and international stock markets including Japanese and European markets. Early studies use linear correlation coefficient as measure of dependence and show that the correlation between stock market returns is not constant over time (Erb et al., 1994; Longin and Solnik, 1995; Ang and Chen, 2002). Other studies apply multivariate time series models like VAR and VECM but find mixed results (for a recent literature review, see Wang, 2014). Recent studies rely on dynamic conditional correlation (DCC) GARCH (Hyde et al., 2007) and regime-switching models (Ang and Timmermann, 2011; Guo et al., 2011), and provide evidence of asymmetric dependence. In particular, they find greater correlation during market downturns than market upturns.

It is a stylized fact that the stock market returns are not normally distributed but characterized by time-varying skewness (Harvey and Siddique, 1999) and time-varying kurtosis (Jondeau and Rockinger, 2003; Brooks et al., 2005). Therefore, the linear correlation coefficient is not suitable for measuring the dependence and can largely lead to misleading results (for more discussions about the bias in the linear correlation coefficient, see Embrechts et al., 1999; Embrechts et al., 2002; Forbes and Rigobon, 2000). In addition, the DCC-GARCH and regime-switching models fail to reproduce asymmetric dependence and do not give information about tail dependence. The tail dependence indicates whether two stock markets crash or boom together. To do so, an alternative approach based on copulas has been adopted. Two main advantages of this approach, first, it can separate the dependence from the marginals without making any assumptions about their distributions. Second, it allows to model directly the tail dependence. The copula functions have been applied by some studies to analyze the dependence between American and developed stock markets\(^1\) (Angel Canela and Pedreira Collazo, 2006; Kole et al., 2005; Hu, 2006; Rodriguez, 2007; de Melo Mendes and Kolev, 2008; Sun et al., 2009; Ignatievay and Platen, 2010; Chollete et al., 2011; Boubaker and Sghaier, 2013).

Although, these studies provide evidence of asymmetric tail dependence, they consider Elliptical and Archimedean copulas which assume that the dependence, i.e., the parameters of the copula and tail dependence are constant. This assumption seems to be restrictive since these parameters vary over time. In particular, they are supposed to increase during financial crisis period due to the existence of contagion effect (Forbes, 2012; Wu et al., 2013).

\(^1\)The copula approach has been also used to study the dependence between American and emerging stock markets (Aloui et al., 2011).
To take into account time-varying dependence structure, Patton (2006) extends the theorem of Sklar (1959) for conditional distributions and proposed a parametric model to describe the evolution of the copula parameter and tail dependence coefficients. Interestingly, the functional form of the copula is assumed to be constant through the period whereas the copula parameter and tail dependence coefficients vary according to some evolution equation.

The time-varying copulas have been adopted by some studies to analyse the dynamic dependence between American and other stock markets. Among these studies, Bartram et al. (2007) focus on the dependence between American and seventeen European stock markets and find strong evidence of time-varying dependence. A similar result is provided by Hu (2010) and Wang et al. (2011) for the dependence between American and Chinese/European stock markets. In the same context, Kenourgios et al. (2011) show that the dependence between American and four emerging stock markets varies also over time.

The objective of this paper is double. First, we examine the dynamic dependence between American and four developed stock markets, namely, Japan, United Kingdom, Germany and France using time-varying copulas. Second, we check whether this dependence differs from Japanese and European markets.

The econometric methodology used is based on two steps. In the first step, we specify the marginal distributions by using the extreme value theory (EVT). However, applying EVT to the returns series is inappropriate since they are not independently and identically distributed. Following McNeil and Frey (2000), we fit the returns series by applying a long memory model to capture the presence of long range dependence in both mean and conditional variance. Then, we employ EVT to the residuals. In the second step, we focus on the dependence. For that, we consider the time-varying Normal copula as benchmark and the time-varying Symmetrized Joe Clayton (SJC) copula. The advantage of the later copula is that it can be either symmetric or asymmetric. As advanced by Kim (2005) and Hu (2010), we do not estimate separately the time-varying Student, Gumbel and Clayton copulas because we do not obtain different results from estimating the time-varying SJC copula alone.

The extreme-value time-varying copula approach has been used by Bhatti and Nguyen (2012) to study the dependence between six stock markets including Australia, Hong Kong, Japan, UK, US and Taiwan. They find evidence of extreme-value time-varying dependence. However, these authors consider a pre-global financial crisis period from April 1993 to October 2007. In this paper, we consider a more recent period from January 1999 to February 2013 that includes the US subprime crisis 2008 and global financial crisis 2007-2009.

Examining the empirical dependence between American and other developed stock markets have important implications for international portfolio diversification and risk management. Indeed, understanding and measuring the dependence between stock market returns can help the investors to select their portfolio and to compute more accurately the extreme risk.

The remainder of this paper is organized as follows. Section 2 presents the extreme-value time-varying copula functions. Section 3 describes the data and provides the empirical results. Section 4 concludes the paper.
2 Extreme-value time-varying copula functions

This section provides the econometric methodology that we adopt to investigate the dependence structure between American and Japanese/European stock market returns. This methodology is based on two steps. First, we focus on modelling the marginal distributions using a combination of long memory model and extreme value theory. Second, we investigate the dependence structure using time-varying copulas.

2.1 Long memory extreme-value model

It is well known that the stock market returns exhibit long memory behavior consisting in long range dependence in both return and conditional volatility. To take into account the presence of such behavior, we use the AutoRegressive Fractionally Integrated Moving Average-Fractionally Integrated Generalized AutoRegressive Conditionally Heteroskedastic, ARFIMA\((p_m, d_m, q_m)\)-FIGARCH\((p_v, d_v, q_v)\), model introduced by Granger and Joyeux (1980) and Baillie et al. (1996). Formally, let \(R_{it}\) denotes the return of stock market \(i\) at time \(t\), we have:

\[
\begin{align*}
\phi(L)(1 - L)^{d_m}(R_{it} - \mu_i) &= \theta(L)\varepsilon_{it}, \quad (1) \\
\varepsilon_{it} &= z_{it}\sigma_{it}, \quad (2) \\
\varpi(L)(1 - L)^{d_v}\varepsilon_{it}^2 &= w_i + (1 - \beta(L))\eta_{it}. \quad (3)
\end{align*}
\]

Equation (1) is the ARFIMA \((p_m, d_m, q_m)\) model. Where \(\mu_i\) is a constant, \(\phi(L)\) and \(\theta(L)\) are polynomials in the lag operator \((L)\) of orders \(p_m\) and \(q_m\) respectively, and \(d_m\) is the fractional integration parameter. When \(0 < d_m < 1/2\), the ARFIMA model exhibit a long memory behaviour. When \(d_m = 0\), it is reduced to an ARMA model and when \(d_m \geq 0.5\) it becomes nonstationary. \(\varepsilon_{it}\) is the independent and identically distributed residuals with zero-mean and time-varying conditional variance \(\sigma_{it}^2\).

Equation (2) defines this residual as a product of positive time-varying function \(\sigma_{it}\) and innovation \(z_{it}\).

Equation (3) represents the FIGARCH \((p_v, d_v, q_v)\) model. Where \(w_i\) is a constant, \(\varpi(L)\) and \(\beta(L)\) are polynomials of degrees \(p_v\) and \(q_v\), and \(0 < d_v < 1\) is the fractional integration parameter. When \(d_v = 0\), the FIGARCH model is reduced to a standard GARCH model and when \(d_v = 1\) it becomes nonstationary IGARCH model.

It is well known that the filtered residuals with ARFIMA-FIGARCH model do not follow a normal distribution. For that, we apply the extreme value theory following the Peaks Over Threshold (POT) method. In particular, we consider the Generalized Pareto Distribution (GDP) distribution in the lower or upper tail and the empirical distribution in the remaining part. The upper and lower thresholds are fixed such as the proportion of excesses represents on each side of
the distribution 10% of the number of observations. Specifically, the distribution of the residuals is followed by:

\[
F(z) = \begin{cases} 
\frac{N_uL}{N} \left( 1 + \xi L \frac{uL - z}{\beta L} \right)^{-\frac{1}{\xi L}} & z < uL \\
\phi(z) & uL < z < uR \\
1 - \frac{N_uR}{N} \left( 1 + \xi R \frac{uR - z}{\beta R} \right)^{-\frac{1}{\xi R}} & uR < z
\end{cases}
\]  
(4)

Where \(u_L\) and \(u_R\) are the upper and lower parameters respectively, \(N_uL\) (resp. \(N_uR\)) represents the number of exceedances over the threshold \(u_L\) (resp. \(u_R\)). \(N\) is the sample. \(\phi(z)\) is the empirical distribution function. \(\beta_L\) and \(\xi_L\) (resp. \(\beta_R\) and \(\xi_R\)) are the scale and shape parameters on the lower (resp. upper) tail.

In order to estimate \(\beta\) and \(\xi\), we have to choose the proper threshold. The choice of the optimal threshold can be tricky because there is a tradeoff between high precision and low variance. If we choose thresholds that are too low, we might obtain biased estimates because the limit theorems do not apply anymore. On the other hand, if we choose high thresholds, it will generate estimates with high standard errors due to limited number of observations. Following Du-Mouchel (1983), we choose the exceedances to be the 10 th percentile of the sample and we use the sample MEF plot and Hill plot to determine an appropriate threshold.

2.2 Time-varying copula functions

The time-varying copulas have been introduced by Patton (2006) to allow for time-variation in the dependence structure. They constitute an extension of Sklar’s theorem to conditional case. In what follows, we give a general definition of the conditional copula and we present the time-varying copula functions used to examine the dependence between US and other developed stock markets.

**Definition of conditional copula:** The conditional copula \(C\) is the joint distribution function of \(F_{X|W}(x|w)\) and \(F_{Y|W}(y|w)\), where \(F_{X|W}\) and \(F_{Y|W}\) are the conditional marginals of \(X\) and \(Y\) given a conditioning variable \(W\).

**Extension of Sklar’s theorem to conditional copula (Patton, 2006):** Let \(F_{XY|W}(x, y|w)\) be the bivariate conditional distribution of \((X, Y) \setminus W = w\) with continuous conditional marginals \(F_{X|W}(x|w)\) and \(F_{Y|W}(y|w)\). Then, there is a unique conditional copula \(C\) such that:

\[
F_{XY|W}(x, y|w) = C \left( F_{X|W}(x|w), F_{Y|W}(y|w) \right| w) .
\]

\(^2\)There are many ways of capturing possible time variation in the dependence structure. In this paper, we assume following Patton (2006) that the functional form of the copula remains fixed over the sample while the parameters vary according to some evolution equation.
2.2.1 Time-varying Normal copula

The Normal copula is the copula of the multivariate normal distribution and is given by:

$$C^N(u, v; \rho) = \Phi^{-1}(u) \Phi^{-1}(v) \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi \sqrt{1-\rho^2}} \exp \left( \frac{2\rho rs - r^2 - s^2}{2(1-\rho^2)} \right) dr ds. \quad (5)$$

Where $\Phi^{-1}(.)$ is the inverse of the standard normal distribution function and $\rho$ is the general linear correlation coefficient. This copula has zero tail dependence $\lambda_U^N = \lambda_L^N = 0$.

In order to allow for time-varying dependence, we assume that the parameter $\rho_t$ evolves according to:

$$\rho_t = \hat{\lambda} \left( \omega^N + \beta^N \rho_{t-1} + \alpha^N \frac{1}{10} \sum_{i=1}^{10} \Phi^{-1}(u_{t-i}) \Phi^{-1}(v_{t-i}) \right). \quad (6)$$

Where $\hat{\lambda} \equiv (1 - \exp(-x))(1 + \exp(-x))^{-1} = \tanh \left( \frac{x}{\hat{\lambda}} \right)$ is the modified logistic transformation needed to maintain $\rho_t$ within the interval $(-1, 1)$ at all times.

Equation (6) reveals that the copula parameter follows an ARMA (1, 10) type process in which the autoregressive term $\beta^N \rho_{t-1}$ captures the persistence effect and the mean of the product of the last 10 observations of the transformed variables $\Phi^{-1}(u_{t-i})$ and $\Phi^{-1}(v_{t-i})$ captures the variation effect in dependence.

2.2.2 Time-varying Symmetrized Joe Clayton copula

The Symmetrized Joe Clayton (SJC) copula is developed by Patton (2006). It constitutes a modification of the Joe-Clayton (JC) copula and can be written as:

$$C^{SJC}(u, v; \tau^U, \tau^L) = 0.5 \left( C^{JC}(u, v; \tau^U, \tau^L) + C^{JC}(1-u, 1-v; \tau^U, \tau^L) + u + v - 1 \right) \quad (7)$$

Where $C^{JC}$ is the Joe-Clayton copula given by $C^{JC}(u, v; \tau^U, \tau^L) = 1 - \left(1 - \left\{[1-(1-u)^\kappa]^{-\gamma} + [1-(1-v)^\kappa]^{-\gamma} - 1\right\}\right)^{1/\kappa}$, $\kappa = 1/\log_2 (2 - \tau^U)$, $\gamma$, $\tau^U$, $\tau^L$ is the upper tail dependence and $\tau^L$ is the lower tail dependence.

Contrary to the Normal copula, the SJC copula can be either symmetric or asymmetric. If $\tau^U = \tau^L$, the dependence is symmetric, otherwise it is asymmetric. Moreover, it captures the lower and the upper tail dependence at the same time.

To allow for time-varying dependence, we specify that $\tau^U_t$ and $\tau^L_t$ vary over time according to:
\[ \tau_i^U = \Lambda \left( \omega^{SJC}_U + \beta^{SJC}_U \tau_{i-1}^U + \alpha^{SJC}_U \frac{1}{10} \sum_{i=1}^{10} |u_{t-i} - v_{t-i}| \right). \] (8)

\[ \tau_i^L = \Lambda \left( \omega^{SJC}_L + \beta^{SJC}_L \tau_{i-1}^L + \alpha^{SJC}_L \frac{1}{10} \sum_{i=1}^{10} |u_{t-i} - v_{t-i}| \right). \] (9)

Where \( \Lambda \equiv (1 + \exp(-x))^{-1} \) is the logistic transformation used to keep \( \tau^U \) and \( \tau^L \) within the interval (0, 1) at all times.

Equations (8) and (9) show that the upper and lower tail dependence parameters follow an ARMA (1, 10) type process in which the autoregressive terms \( \beta^{SJC}_L \tau_{i-1}^L \) and \( \beta^{SJC}_U \tau_{i-1}^U \) capture the persistence effect and the forcing variables represented by the mean absolute difference between \( u_t \) and \( v_t \) over the previous 10 observations captures the variation effect in dependence.

To estimate the parameter \( \phi = (\omega, \beta, \gamma) \) of the time-varying copula, we use the Canonical Maximum Likelihood (CML) method. Compared to the Full Maximum Likelihood (FML) and the Inference Function for Margins (IFM) estimation methods, no assumptions about marginal distributions are needed to estimate the parameter of the time-varying copula. With CML, we perform two estimation steps. The first one consists on using the empirical distribution of \( x_t \) and \( y_t \) to transform them into \( \hat{u}_t \) and \( \hat{v}_t \). The second consists on applying Maximum Likelihood (ML) to estimate the parameter \( \phi \) by solving the following problem:

\[ \hat{\phi} = \arg \max_{\phi} \sum_{i=1}^{T} \ln c(\hat{u}_t, \hat{v}_t; \phi). \] (10)

Where \( \hat{u}_t = F_X(x_t; \hat{\phi}_x) \) and \( \hat{v}_t = F_Y(y_t; \hat{\phi}_y) \) are pseudo-sample observations from the copula.

To choose the best fitting copula, we consider the log likelihood (LL) and two information criteria, namely, Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). In addition, we apply the likelihood ratio (LR(\( p \))) test which tests the null hypothesis of constant dependence (restricted copula) against the alternative of time-varying dependence (unrestricted copula), where \( p \) is the number of restrictions under \( H_0 \). So, we have two restrictions in Normal copula and four restrictions in SJC copula.

3 Empirical results

This section gives an empirical study of the dependence between the American and four developed stock markets including Japan, United Kingdom (UK), Germany and France. First, we describe the data. Then, we present the obtained results.
3.1 Data description

Our data consist of five daily stock market indices containing Dow Jones (US), Nikkei 225 (Japan), FTSE 100 (UK), DAX 30 (Germany) and CAC 40 (France) from January 1, 1999 through February 22, 2013, for a total of 3692 observations. These data are obtained from Datastream. They are transformed in logarithm form and considered in first difference. So, the series obtained correspond to stock market returns\(^3\). The obtained results for unit root tests without and with structural breaks indicate that there is no evidence of unit root and that the returns series follow a stationary process\(^4\). Figure 1 plots the stock market indices and Table 1 summarises descriptive statistics for each stock market return.

Table 1. Summary statistics for each stock market return

<table>
<thead>
<tr>
<th></th>
<th>(R_{US})</th>
<th>(R_{JP})</th>
<th>(R_{UK})</th>
<th>(R_{GN})</th>
<th>(R_{FR})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.000</td>
<td>-5.2938 e-5</td>
<td>2.0109 e-5</td>
<td>0.108</td>
<td>-1.6755 e-5</td>
</tr>
<tr>
<td>Std.dev</td>
<td>0.012</td>
<td>0.015</td>
<td>0.013</td>
<td>0.016</td>
<td>0.016</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.048</td>
<td>-0.387</td>
<td>-0.146</td>
<td>-0.002</td>
<td>0.025</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>7.622</td>
<td>7.095</td>
<td>5.830</td>
<td>4.218</td>
<td>4.674</td>
</tr>
<tr>
<td>JB</td>
<td>8934.5***</td>
<td>7832.8***</td>
<td>5239.3***</td>
<td>2735.0***</td>
<td>3359.3***</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
</tr>
<tr>
<td>LB (10)</td>
<td>47.027***</td>
<td>14.546*</td>
<td>85.362***</td>
<td>21.884***</td>
<td>55.245**</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.105]</td>
<td>[0.000]</td>
<td>[0.016]</td>
<td>[0.034]</td>
</tr>
<tr>
<td>ARCH (10)</td>
<td>111.460***</td>
<td>152.68***</td>
<td>108.02***</td>
<td>80.864***</td>
<td>76.946***</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
</tr>
</tbody>
</table>

Notes: \(R_{US}\), \(R_{JP}\), \(R_{UK}\), \(R_{GN}\) and \(R_{FR}\) are stock market returns of United States, Japan, United-Kingdom, Germany and France. JB is the statistic of Jarque and Bera test for normality, LB (10) is the statistic of Ljung-Box test with 10 lags for serial correlation and ARCH (10) is the statistic of ARCH test with 10 lags for conditional heteroskedasticity. \(\rho\) is the linear correlation coefficient between US and other stock market returns. The numbers in brackets are p-values. ***, ** and * indicate a rejection of the null hypothesis at the 1%, 5% and 10% levels respectively.

The mean of the stock market returns are close to zero in most of the cases. The German stock market exhibits the biggest average return and documents the largest volatility as measured by the standard deviation (1.6%), reflecting the fact that the higher return tend to compensate the higher risk. All stock markets experience negative skewness (except France) and show excess of kurtosis. The results of the Jarque-Bera test strongly reject the null hypothesis of normality for all return series. The Ljung-Box test shows significant evidence of serial correlation for all returns series and the ARCH test indicates presence of heteroscedasticity in all return series. In addition, a preliminary analysis of the autocorrelations functions of the returns and squared returns returns show

\(^3\)More precisely, we consider stock market returns \(r_t\) defined as \(r_t = \ln P_t - \ln P_{t-1}\), where \(P_t\) is the stock market index.

\(^4\)The results of the unit root tests are not reported here but are available upon request.
evidence of a slow decay at the hyperbolic rate\(^5\), which justify the choice of the AFIMA-FIGARCH to model the marginal distributions.

### 3.2 Marginal distributions

In a first step, we fit each stock market return by applying the ARFIMA-FIGARCH model given by Equations (1) and (3). The parameters of the model are estimated using the quasi-maximum likelihood estimation method. The estimation results are reported in Table 2.

<table>
<thead>
<tr>
<th>((p_m, d_m, q_m))</th>
<th>(R_{US})</th>
<th>(R_{JP})</th>
<th>(R_{UK})</th>
<th>(R_{FR})</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0, d_m, 0))</td>
<td>((0, 0, 0))</td>
<td>((0, d_m, 0))</td>
<td>((0, 0, 0))</td>
<td>((1, d_m, 0))</td>
</tr>
<tr>
<td>((1, d_v, 1))</td>
<td>((1, d_v, 1))</td>
<td>((1, d_v, 1))</td>
<td>((1, d_v, 1))</td>
<td>((1, d_v, 1))</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(\mu)</th>
<th>(d_m)</th>
<th>(\phi_1)</th>
<th>(\theta_1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d_m)</td>
<td>0.068***</td>
<td>0.051***</td>
<td>0.090***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.709)</td>
<td>(3.797)</td>
<td>(3.875)</td>
<td></td>
</tr>
<tr>
<td>(\phi_1)</td>
<td>0.053**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.917)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(w \times 10^4)</td>
<td>0.062***</td>
<td>0.026***</td>
<td>0.046***</td>
<td>0.044***</td>
</tr>
<tr>
<td></td>
<td>(2.679)</td>
<td>(2.523)</td>
<td>(2.857)</td>
<td>(2.692)</td>
</tr>
<tr>
<td>(d_v)</td>
<td>0.467***</td>
<td>0.522***</td>
<td>0.539**</td>
<td>0.629***</td>
</tr>
<tr>
<td>(\beta_1)</td>
<td>0.465***</td>
<td>0.585***</td>
<td>0.059***</td>
<td>0.639***</td>
</tr>
<tr>
<td></td>
<td>(3.044)</td>
<td>(4.945)</td>
<td>(4.329)</td>
<td>(7.399)</td>
</tr>
<tr>
<td>(\omega_1)</td>
<td>0.191***</td>
<td>0.124***</td>
<td>0.542**</td>
<td>0.028***</td>
</tr>
<tr>
<td></td>
<td>(3.040)</td>
<td>(2.120)</td>
<td>(7.775)</td>
<td>(2.318)</td>
</tr>
</tbody>
</table>

| Skw | 0.387 | -0.324 | -0.462 | -0.270 | -0.282 |
| Ex. Kurt | 2.635 | 1.394 | 2.474 | 1.090 | 0.840 |
| \(Q^2(20)\) | 10.830 | 20.289 | 15.558 | 22.446 | 19.703 |

Notes: The values in parenthesis are the t-Student. Skw is the skewness of the standardized residuals. Ex. Kurt is the excess of kurtosis of the standardized residuals. \(Q(20)\) is the Ljung-Box statistic for serial correlation in returns for order 20. \(Q^2(20)\) is the Ljung-Box statistic for serial correlation in squared returns for order 20. *, ** and *** denote significance at the 10%, 5% and 1% levels respectively.

We note that the parameter \(d_m\) is positive, significant and less than \(1/2\) for the American, British and French stock market returns, indicating a long range dependence in the mean. The parameter \(d_v\) is positive and significant for all stock market returns, reflecting a finite persistence in the conditional

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\(^5\)Here, we do not report the autocorrelation functions of the returns and squared returns. These are available upon request.
volatility. The Ljung-Box test applied to the returns and squared returns show the absence of serial correlation. Therefore, the ARFIMA-FIGARCH seems to be adequate. However, the descriptive statistics of the standardized residuals indicate clearly that the conditional distribution has a heavier tail than that of a normal distribution. As described in Section 2.1, we employ the POT method using the GPD for the lower and upper tails and the empirical CDF for the interior estimation (Equation (5)). We choose the exceedances to be the 10th percentile of the sample.

### 3.3 Dynamic dependence structure

For each US related pair, we estimate the time-varying Normal and SJC copulas described in Section 2.2. The obtained results are reported in Table 3.

<table>
<thead>
<tr>
<th>Table 3. Estimated parameters for time-varying copulas</th>
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</thead>
<tbody>
<tr>
<td>((R_{US}, R_{JP}))</td>
</tr>
<tr>
<td><strong>Time-varying Normal copula</strong></td>
</tr>
<tr>
<td>(\omega^N)</td>
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<tr>
<td></td>
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<tr>
<td>(\beta^N)</td>
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<td></td>
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<td>(\gamma^N)</td>
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<tr>
<td><strong>LL</strong></td>
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<tr>
<td><strong>AIC</strong></td>
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<tr>
<td><strong>BIC</strong></td>
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<tr>
<td><strong>Time-varying SJC copula</strong></td>
</tr>
<tr>
<td>(\omega^SJC_U)</td>
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<tr>
<td></td>
</tr>
<tr>
<td>(\beta^SJC_U)</td>
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<td></td>
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<tr>
<td>(\omega^SJC_L)</td>
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<td>(\beta^SJC_L)</td>
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<td>(\gamma^SJC_L)</td>
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<td><strong>AIC</strong></td>
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<td><strong>BIC</strong></td>
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</tbody>
</table>

Notes: The parameters \(\omega^N\), \(\beta^N\), and \(\gamma^N\) are given by Equation (6). The parameters \(\omega^SJC_U\), \(\beta^SJC_U\), and \(\gamma^SJC_U\) are given by Equation (8). The parameters \(\omega^SJC_L\), \(\beta^SJC_L\), and \(\gamma^SJC_L\) are given by Equation (9). The numbers in parentheses are t-Student. ***, ** and * indicate statistical significance at 1%, 5% and 10% levels respectively.
We find that the dependence between American and Japanese stock markets is described by the time-varying Normal copula since it exhibits the smallest LL⁶, AIC and BIC while that of American and European stock markets is rather represented by the time-varying SJC copula. This choice is confirmed by a the likelihood ratio test⁷. This result suggests evidence of symmetric dynamic dependence between American and Japanese stock markets. In contrast, the dynamic dependence between American and European stock markets is asymmetric. These findings are consistent with previous studies like Hu (2006) and Bhatti and Nguyen (2012) who find that the time-varying Normal copula gives a better fit for the dependence between American and Japanese stock markets whereas the time-varying SJC copula performs better the dependence between American and European stock markets.

Concerning the time-varying SJC copula’s dependence, the parameters β_{SJC}^L and β_{SJC}^U represents the degree of persistence while α_{SJC}^L and α_{SJC}^U captures the adjustment in the dependence process. β_{SJC}^L is negative and significant in the pair \((R_{US}, R_{UK})\) only and β_{SJC}^U is negative and significant in the pairs \((R_{US}, R_{GN})\) and \((R_{US}, R_{FR})\). α_{SJC}^L and α_{SJC}^U is negative and significant in all pairs. This result indicates a high persistence in the dependence level and confirms that the dependence is time-varying and that a constant copula model may be not adequate for describing the dependence between the American and other stock markets considered.

Regarding the conditional tail dependences ω_{SJC}^L and ω_{SJC}^U, we see that for the pairs \((R_{US}, R_{GN})\) and \((R_{US}, R_{FR})\), the conditional upper tail dependence ω_{SJC}^U is positive and significant while the conditional lower tail dependence ω_{SJC}^L is not significant, implying that there is a higher possibility of joint extreme events during bull markets rather than bear markets. This indicate that when the market is up, the risk diversification is less effective due to this greater dependence. In particular, the american stock market exhibits greater dependence during market downturns with German than French stock market. This could be attributable to the fact that America’s more frequent trading with Germany which in turn may be caused by the geographical proximity and regional economic developments. As indicated in Table 1, the German and French stock markets exhibit the same volatility level (16%) but the German stock market return displays higher average return (0.012%) than the French stock market return (-0.002%), reflecting thus a better stock market performance. For the pair \((R_{US}, R_{UK})\), ω_{SJC}^U is not significant whereas ω_{SJC}^L is positive and significant, indicating more joint positive than negative extremes.

These findings have important implications for international portfolio diversification and risk management. Indeed, the presence of higher dependence between American and other stock market market returns implies a limited opportunity for portfolio diversification. Managers of portfolios that uses VaR or

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⁶We note that the selected copula in terms of LL is one with lowest LL, since we minimize in our estimation (−LL) rather maximize (LL).

⁷In practice, for each pair, we estimate two alternatives specifications of copulas (constant and time-varying). For all pairs, the null hypothesis of constant copula is rejected against time-varying copula. Here, we do not report the results. These are available upon request.
other downside risk measures should emphasize the left side of portfolio return distribution.

Consequently, when the American stock market is down, the investor should include German or French stocks to gain benefits from diversification. In contrast, when the American stock market is up, the investor should rather incorporate British stocks. Further, to reduce the risk, US investor should include Japanese stocks.

Figure 2 displays the time paths of the conditional Normal parameter for the pair \((R_{US}, R_{JP})\). Figures 3-5 plot the time paths of the conditional lower and upper tail dependences for the pairs \((R_{US}, R_{UK}), (R_{US}, R_{GN})\) and \((R_{US}, R_{FR})\) respectively. For the pair \((R_{US}, R_{UK})\), we see that the conditional lower tail dependence is greater than the conditional upper tail dependence while for the pairs \((R_{US}, R_{GN})\) and \((R_{US}, R_{FR})\), we observe that the conditional upper tail dependence is greater than conditional lower tail dependence, supporting evidence of asymmetry in tail dependencies.

4 Conclusion

In this paper, we examine the dynamic dependence structure between American stock markets and Japanese/three European stock markets using an extreme-value time-varying copula approach. First, we estimate an ARFIMA-FIGARCH model for each stock market return and we find that the conditional distribution of the standardized residuals present a heavier tail than that of Normal distribution. Thus, we consider the Generalized Pareto Distribution and we choose the exceedances to be the 10th percentile of the sample. Then, we focus on the conditional dependence using time-varying Normal and SJC copulas. The empirical results show that for all pairs except that of (US, Japan), we find strong evidence of asymmetric dynamic dependence while the dynamic dependence between American and Japanese stock markets is symmetric. In particular, the American and Euro-European (German and French) stock markets are more dependent during bull markets while the American and Non Euro-European (British) stock markets are more dependent during bull markets. These findings can help investors to better diversify the portfolio and manage the risk.
References


Figure 1: Evolution of the five stock market indices
Figure 2: Time paths of time varying Normal copula parameter for (US-JP)

Figure 3: Time paths of time varying lower and upper tail dependence for (US-UK)
Figure 4: Time paths of time varying lower and upper tail dependence for (US-GN)

Figure 5: Time paths of time varying lower and upper tail dependence (US-FR)