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Modelling Return and Volatility of Oil Price using Dual Long Memory Models

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Abstract

This paper investigates the dynamic properties of both return and volatility of the oil price. The analysis is carried out using a set of double long memory specifications incorporating several features such as long range dependence, asymmetry in conditional variances and time varying correlations. The in-sample diagnostic tests as well as the out-of-sample forecasting results show the performance of the ARFIMA-FIAPARCH model.

Keywords: Oil price, return, volatility, dual long memory.
1. Introduction

Modelling the dynamics of oil price has been extensively examined in literature. Some authors focus on the return while others authors are interested on the volatility. For that numerous models have been applied. Among these models, the authors use the long memory models.

The long range dependence phenomenon has raised a challenging problem in financial time series analysis and has been the subject of an extensive theoretical and empirical investigation. The presence of long memory components in the generating mechanism of oil price is a key issue which has important implications for risk management and portfolio allocation.

The long memory property describes the high-order correlation structure of a given time series. If a series exhibits long memory, there is persistent temporal dependence even between distant observations. Such series are characterized by a slowly decaying autocovariance function along with an unbounded spectral density function at the null frequency.

To account for this typical behaviour, Granger and Joyeux (1980) and Hosking (1981) introduced the Autoregressive Fractionally Integrated Moving Average (ARFIMA) processes. The ARFIMA model captures the long term dependence pattern by allowing the integration order of the conventional ARIMA models to take non-integer values.

Besides looking at the conditional mean of a time series, long memory effects in the volatility process have also been widely investigated. Indeed, Baillie et al. (1996) proposed the Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity (FIGARCH) model which combines high temporal dependencies in the second conditional moments with the virtues of a parsimonious parameterization. Since then, the intuitive concept of fractional unit root in the variance has been extended to other GARCH type specifications resulting in a collection of long memory adaptations such as the Fractionally Integrated Exponential GARCH (FIEGARCH) of Bollerslev and Mikkelsen (1996), the Fractionally Integrated Asymmetric Power ARCH (FIAPARCH) of Tse (1998) and the Hyperbolic GARCH (HYGARCH) of Davidson (2004).

Until recently, empirical research has been focusing on long memory dynamics in either the conditional mean or in the conditional variance of financial time series. In a pioneering work, Teyssière (1997) introduced the double long memory ARFIMA-FIGARCH model which generates long range dependencies in both the first and the second conditional moments. The hybrid specification is more than a simple juxtaposition of two fractional processes, in the sense
that the joint estimation of the ARFIMA and FIGARCH components respectively in the mean
and in the variance equations proves to be crucial for estimation and forecasting issues.

This paper examines the presence of fractional dynamics in both return and volatility of oil
price. A set of dual long memory models reproducing an assortment of stylized features is used to
fit the dynamic structure of the analyzed series.

The remainder of the paper is structured as follows. Section 2 describes the dual long memory
used. Section 3 presents the empirical results. Section 4 provides a comparative study of the out-
of-sample forecasting performances of the selected models. Section 5 concludes the paper.

2. Dual long memory models

In this section, we present a collection of dual long memory specifications that we have
used to model the dynamics of the return and the volatility of the oil price. More precisely, we use the
ARFIMA-FIGARCH, the ARFIMA-HYGARCH and the ARFIMA-FIAPARCH models.

2.1. ARFIMA-FIGARCH model

The double long memory ARFIMA-FIGARCH process consists in modelling the analysed
series by inserting fractional filters in both the mean and the variance equations. In the
conditional mean, we fit a ARFIMA\((p, d, q)\) specification given formally by:

\[
\theta(L)(1-L)^{d_w}(r_\mu) = \phi(L)\varepsilon_t, \quad \varepsilon_t | \Psi_{t-1} \sim D(0, h_t),
\]

where \(\mu\) is the mean of the process, \(d_w\) is a fractional number, \(\theta(L) = 1-\theta_1L-\cdots-\theta_pL^p\) and
\(\phi(L) = 1+\phi_1L+\cdots+\phi_qL^q\) are the AR and MA polynomials in the lag operator of respective orders
\(p\) and \(q\) (with all roots lying outside the unit circle), \(\Psi_{t-1}\) stands for the information set available
at time \(t-1\) whereas the residuals are assumed to follow the conditional distribution \(D\).

In the variance equation, we retain a FIGARCH\((P, \delta, Q)\)-type adaptation:

\[
h_t = \sigma + \left\{1 - (1-L)^{d_e}\sigma(L)(1-L)^{d_e}\right\}\varepsilon_t^2,
\]

where \(h_t\) is the conditional variance of \(r_t\), \(\sigma\) is the mean of the process, \(d_e\) is the fractional degree
of integration of \(h_t\) and \(\beta(L)\) and \(\sigma(L)\) are lag polynomials of respective orders \(P\) and \(Q\). An
interesting feature of the FIGARCH model is that it nests both the GARCH model (Bollerslev
(1986)) for \(d_e = 0\), and the IGARCH specification (Engle and Bollerslev (1986)) for \(d_e = 1\). In the
first case, shocks to the conditional variance decay at an exponential rate with the lag length while in the second, shocks remain important for all forecast horizons thus revealing an infinite persistence behaviour. If $0 < d_v < 1$, there is long term dependence in the conditional variance indicated by a hyperbolic decay of the autocorrelation and autocovariance functions.

It is noteworthy that FIGARCH-type processes, although strictly stationary and ergodic for $d_v \in [0,1]$, are not covariance stationary. Furthermore, the interpretation of the long memory parameter $\delta$ is difficult in the FIGARCH set up (see Davidson (2004) for additional details).

### 2.2. ARFIMA-HYGARCH model

To cope with the deficiencies inherent to the FIGARCH framework, Davidson (2004) introduced a more general class of long-memory GARCH processes, called hyperbolic GARCH (HYGARCH). These processes allow for a faster non-geometric (hyperbolic) rate of decay for which covariance stationarity would still be achievable.

The ARFIMA-HYGARCH model adopts equation (1) in the first conditional moment whilst the conditional volatility is modelled using a HYGARCH parameterization given formally by:

$$
\sigma_t^2 = \alpha + \left[\left(1-(1-\beta(L))^{-1}\right)\sigma(L)\left[1+\alpha\left((1-L)^{d_v} - 1\right)\right]\right] \varepsilon_t^2
$$

The parameters $\alpha$ and $d_v$ are assumed to be non-negative. Under the condition $\alpha < 1$ and if the GARCH component obeys the usual covariance stationarity restrictions (Bollerslev (1986)), the resulting stochastic process is weakly stationary (see Davidson (2004) for further details).

The HYGARCH model nests the FIGARCH process for $\alpha = 1$ and the stable GARCH process for $\alpha = 0$. It is notable here that in the latter case, the fractional parameter $d_v$ is unidentified thus raising a problem in constructing hypothesis tests. Davidson (2004) stated that when $d_v = 1$, the parameter $\alpha$ reduces to an autoregressive root reproducing geometric memory cases, namely GARCH models for $\alpha < 1$ and Integrated GARCH specifications for $\alpha = 1$. Hence testing the restriction $d_v = 1$ allows to discriminate between geometric memory and hyperbolic memory dynamics. In that case, the GARCH or IGARCH type specifications will correspond to $\alpha < 1$ and $\alpha = 1$ respectively.
2.3. ARFIMA-FIAPARCH model

It should be noticed that the models introduced above i.e. FIGARCH and HYGARCH disregard an important stylized fact inherent to financial markets: the so-called “leverage effect” (Black (1976)) which corresponds to negative correlations between past returns and future volatility.

To take into account asymmetric responses of volatility to positive and negative shocks along with volatility persistence behavior, Tse (1998) extended the Asymmetric Power GARCH model of Ding, Granger and Engle (1993) by incorporating a fractional filter in the conditional variance equation. The obtained model is known as FIAPARCH.

In the following, we present the ARFIMA-FIAPARCH model which generates long memory properties in both the first and the (power transformed) second conditional moments. In this dual long memory framework, the conditional mean is fitted by an ARFIMA-type adaptation (equation (1)) whereas the conditional variance equation is expressed as a power transformation of the standard deviation and is given formally by:

\[ h_{t}^{\delta} = \omega + \left\{1 - (1 - \beta(L))^{-1} \sigma(L)(1-L)^{d}\right\} \left(|\varepsilon| - \gamma \varepsilon\right)^{\delta} \]

where \(-1 < \gamma < 1\) and \(\delta > 0\). Here the power term \(\delta\) plays the role of a Box-Cox transformation of the conditional standard deviation \(h_{t}^{\delta}\) while \(\gamma\) denotes the asymmetry coefficient accounting for the leverage effect.

When \(\gamma > 0\), negative shocks give rise to higher volatility than positive shocks. The reverse applies if \(\gamma < 0\); the magnitude of the shocks being captured by the term \(|\varepsilon| - \gamma \varepsilon\).

It is noteworthy here that the use of the power term \(\delta\) is an endeavour to go beyond the Gaussianity assumption. In fact, if the datasets are assumed to follow a conditional normal density, then the first two moments, (i.e. the mean and the variance), completely typify the distribution of returns. This justifies the common use of a squared term \(\delta=2\) and hence a measure of the variance to characterize the volatility structure. However, since asymmetry and heavy tails are both stylized facts of financial asset returns, the hypothesis of normality seems to be unrealistic and higher order moments such as Skewness and Kurtosis are required to specify the true underlying distribution. In such a context, considering the variance as a measure of the volatility process (i.e. setting \(\delta=2\)) can adversely affect the estimation results and the forecasting
performances of our models. To deal with this issue, Ding, Granger and Engle (1993) suggest estimating the volatility measure in the form of a power transformation through allowing an optimal power term $\delta$ to be endogenized and freely determined from the data.

The FIAPARCH process couples the flexibility of a varying exponent with the asymmetry coefficient thus capturing asymmetric volatility structure and letting the data determine the power of the heteroscedastic equation. Moreover, it enhances the long memory aspect of the conditional volatility via the fractional differencing parameter $d$. The FIAPARCH process reduces to the FIGARCH process when $\gamma = 0$ and $\delta = 2$.

3. Empirical results

3.1 Description of data

Our database consists of daily oil prices over the period June 1, 2006 until May 25, 2013, yielding a total of 1989 observations. As a proxy for oil price, we use Brent crude oil price collected from the Energy Information Administration.

The data are transformed in logarithm form and are considered in first difference, so the series obtained correspond to oil price changes. The application of standard unit root tests and unit root tests with structural breaks show evidence of stationarity. The results of the unit root tests are not reported here but are available upon request. The descriptive statistics of the oil price changes are presented in Table 1.

<table>
<thead>
<tr>
<th>Table 1: Summary statistics of oil price change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Std. Dev</td>
</tr>
<tr>
<td>Skewness</td>
</tr>
<tr>
<td>Kurtosis</td>
</tr>
<tr>
<td>Jarque-Bera</td>
</tr>
<tr>
<td>Observations</td>
</tr>
</tbody>
</table>

Measures of excess kurtosis indicate that the distributions of returns are leptokurtic and then heavy tailed with significant kurtosis values greater than 3. Besides, the skewness coefficients show that the data are slightly positively skewed with respect to the Gaussian distribution. The Jarque-Bera test statistics are highly significant at 1% significance level thus rejecting the null hypothesis of normality.
3.2 Modelling oil price using dual long memory models

In this section, we estimate the three dual long memory processes (ARFIMA-FIGARCH, ARFIMA-HYGARCH and ARFIMA-FIAPARCH) relying on the Quasi Maximum Likelihood (QML) procedure. In view of the distributional properties of the oil price change, the Student’s t distribution\(^1\) is assumed for the innovations as suggested by Bollerslev (1987).

We note that lag order selection is a vital issue when specifying a dynamic model. In order to identify the truncation orders \(p, q, P, Q\) of the short memory polynomials of the dual long memory adaptations, we use the Schwarz and the Hannan-Quinn Information criteria.

The estimation results, reported in Table 2, show that the fractional parameters in the mean equation of the three dual long memory specifications are highly significant (at the 1% level). This confirms that the oil price returns exhibit a long run dependence structure.

In addition, the fractional orders of integration \(d_v\) in the second conditional moments are statistically significant at the 1% level, which means that the volatility is fractionally integrated. However, it should be stressed here that the three processes don’t share the same degree of fractional integration in their scedastic functions. In fact, estimates of the long memory parameters vary from 0.347 for the HYGARCH adaptation to 0.348 for the FIAPARCH process whereas interestingly the FIGARCH model displays the highest persistence degree with a fractional differencing parameter of 0.425.

The estimated parameters of the ARFIMA-FIGARCH specification show that the fractional order of integration \(d_v\) is highly significant and statistically different from unity and zero (at 1% significance level). This indicates that the impact of shocks to the conditional volatility flaunt a hyperbolic rate of decay as opposed to the conventional exponential decay inherent to the stable GARCH process or the infinite persistence pattern distinguishing IGARCH type models.

Focusing on the estimates of the ARFIMA-HYGARCH process, we observe that the hyperbolic memory in variance measured by the fractional order of integration \(d_v\) is quite pronounced. The amplitude parameter \(\alpha\) is statistically different from 1 leading to the rejection of the Fractionally Integrated GARCH alternative. A noteworthy feature here is the largest value of the \(\alpha\) parameter which exceeds one (\(\alpha>1\)). This suggests that the driving process of the BVMT

\(^1\) We have also estimated the models based on the skewed student distribution. Yet, the obtained results emphasize the superiority of the student’s t distribution. For the sake of conciseness, we restrict ourselves to the latter distribution. Complete results are available upon request.
stock returns is not covariance stationary. It should be stressed that within the ARFIMA-HYGARCH framework, we can test for the restriction embodied in the ARFIMA-FIGARCH model i.e. \( \alpha = 1 \) relying on a likelihood ratio type test. Formally, the likelihood ratio (LR) test is a statistical test used to compare the in-sample performance of nested models. The test statistic is asymptotically chi-squared distributed with degrees of freedom equal to the number of restrictions being tested. If \( \ell_n \) denotes the log-likelihood value under the null hypothesis that the true model is FIGARCH and \( \ell \) is the log-likelihood under the alternative that the true model is HYGARCH, the test statistic \( LR = 2(\ell - \ell_n) \) should follow a \( \chi^2 \) distribution with 1 degree of freedom (the number of restrictions) under the null hypothesis. In this case, \( LR = 18.088 \) thus rejecting the constraint implied by the FIGARCH adaptation \( (\alpha = 1) \) at 1% significance level. Note that the 0.01 critical value of the Chi-square distribution with 1 degree of freedom is \( \chi^2_{(1)} = 6.634 \).

The analysis of the estimation results of the ARFIMA-FIAPARCH parameterization calls for several observations: the power term \( \hat{\delta} \) is statistically different from two for the FIAPARCH parameterization whereas the estimated asymmetry coefficient \( \hat{\gamma} \), although small-valued, is significant and negative implying that positive shocks predict higher volatility than negative shocks. In other words, the negative sign of \( \hat{\gamma} \) suggests that “good news,” increase is more destabilizing than “bad news,” i.e. an unanticipated drop. A Likelihood Ratio tests can be constructed in which the restricted case is the ARFIMA-FIGARCH specification \( (\delta = 2 \text{ and } \gamma = 0) \). The test statistic which is asymptotically \( \chi^2 \) distributed with 2 degrees of freedom (when the null hypothesis is true) yields a value of 21.056 and then clearly rejects the constraints implied by the FIGARCH-type adaptation at 1% significance level \( (\chi^2_{(2)} = 9.210) \).

It is worth mentioning that the FIAPARCH specification adapts particularly well to the conditional variance, since the Box-Pierce test statistics carried out on the squared residuals are smaller than those obtained for the ARFIMA-FIGARCH and ARFIMA-HYGARCH specifications. In addition and while assessing the adequacy of the three dual long memory models, we see that oil price series under the ARFIMA-FIAPARCH structure displays the highest log-likelihood value and the lowest Akaike Information Criteria (AIC) among all competing models.
Table 2: Estimation results for oil price change

<table>
<thead>
<tr>
<th></th>
<th>ARFIMA-FIGARCH</th>
<th>ARFIMA-HYGARCH</th>
<th>ARFIMA-FIAPARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(p,d_w,q)$</td>
<td>(1,d,1)</td>
<td>(1,d,1)</td>
<td>(1,d,1)</td>
</tr>
<tr>
<td>$(P,d,Y)$</td>
<td>(1,δ,1)</td>
<td>(1,δ,1)</td>
<td>(1,δ,1)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>(0.389)</td>
<td>(0.744)</td>
<td>(0.572)</td>
</tr>
<tr>
<td>$d_w$</td>
<td>0.184</td>
<td>0.178</td>
<td>0.172</td>
</tr>
<tr>
<td>$d_Y$</td>
<td>(6.203)**</td>
<td>(6.751)**</td>
<td>(7.006)**</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>-0.208</td>
<td>-0.234</td>
<td>-0.216</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>(-1.742)*</td>
<td>(-2.162)**</td>
<td>(-1.901)**</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>0.279</td>
<td>0.304</td>
<td>0.283</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>(2.230)**</td>
<td>(6.751)**</td>
<td>(2.386)**</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.000</td>
<td>0.001</td>
<td>0.003</td>
</tr>
<tr>
<td>$d_e$</td>
<td>(0.043)</td>
<td>(0.032)</td>
<td>(0.641)</td>
</tr>
<tr>
<td></td>
<td>0.425</td>
<td>0.347</td>
<td>0.348</td>
</tr>
<tr>
<td></td>
<td>(7.886)**</td>
<td>(4.551)**</td>
<td>(6.070)**</td>
</tr>
<tr>
<td>$Log(\alpha)$</td>
<td>-</td>
<td>0.197</td>
<td>-</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>-</td>
<td>-</td>
<td>-0.182</td>
</tr>
<tr>
<td>$\delta$</td>
<td>-</td>
<td>-</td>
<td>(-3.363)**</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.677</td>
<td>0.408</td>
<td>0.648</td>
</tr>
<tr>
<td></td>
<td>(5.421)**</td>
<td>(2.947)**</td>
<td>(6.169)**</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.461</td>
<td>0.293</td>
<td>0.701</td>
</tr>
<tr>
<td></td>
<td>(3.125)**</td>
<td>(2.938)**</td>
<td>(10.670)**</td>
</tr>
<tr>
<td>$\hat{\nu}$</td>
<td>5.131</td>
<td>4.178</td>
<td>4.535</td>
</tr>
<tr>
<td></td>
<td>(17.811)**</td>
<td>(13.901)**</td>
<td>(15.150)**</td>
</tr>
<tr>
<td>$\tau_1$</td>
<td>0.254</td>
<td>0.123</td>
<td>0.084</td>
</tr>
<tr>
<td></td>
<td>(3.635)**</td>
<td>(1.768)*</td>
<td>(1.212)</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>3.999</td>
<td>3.628</td>
<td>3.066</td>
</tr>
<tr>
<td></td>
<td>(28.646)**</td>
<td>(25.984)**</td>
<td>(28.404)**</td>
</tr>
<tr>
<td>$Q(20)$</td>
<td>22.559</td>
<td>22.600</td>
<td>17.359</td>
</tr>
<tr>
<td>$Q^2(20)$</td>
<td>10.365</td>
<td>11.113</td>
<td>9.105</td>
</tr>
<tr>
<td>$BDS(5)$</td>
<td>5.143</td>
<td>4.587</td>
<td>2.987</td>
</tr>
<tr>
<td>Log-Likelihood</td>
<td>1039.039</td>
<td>1048.083</td>
<td>1049.567</td>
</tr>
<tr>
<td>Akaike</td>
<td>-0.072</td>
<td>-0.085</td>
<td>-0.095</td>
</tr>
</tbody>
</table>

Note: ***, ** and * denote significance at 1%, 5% and 10% level respectively.
4. Predictive performance of the dual long memory processes

Now, we evaluate at the end of this article the out-of-sample forecasts of the oil price change. The forecast horizons considered in this study correspond to one, five, ten and fifteen steps ahead (s=1, 5, 10).

Tables and report the out-of-sample forecast evaluation results of the ARFIMA-FIGARCH, the ARFIMA-HYGARCH and the ARFIMA-FIAPARCH models using three evaluation criteria (the Mean Square Error (MSE), the Mean Absolute Prediction Error (MAPE) expressed as a percentage and the Logarithmic Loss (LL) function).

<table>
<thead>
<tr>
<th>Model</th>
<th>Criterion</th>
<th>s = 1</th>
<th>s = 5</th>
<th>s = 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARFIMA-FIGARCH</td>
<td>MSE</td>
<td>0.024</td>
<td>0.016</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>MAPE</td>
<td>0.156</td>
<td>0.125</td>
<td>0.097</td>
</tr>
<tr>
<td></td>
<td>LL</td>
<td>22.035</td>
<td>11.831</td>
<td>11.640</td>
</tr>
<tr>
<td>ARFIMA-HYGARCH</td>
<td>MSE</td>
<td>0.018</td>
<td>0.015</td>
<td>0.011</td>
</tr>
<tr>
<td></td>
<td>MAPE</td>
<td>0.144</td>
<td>0.119</td>
<td>0.087</td>
</tr>
<tr>
<td></td>
<td>LL</td>
<td>19.604</td>
<td>10.807</td>
<td>9.865</td>
</tr>
<tr>
<td>ARFIMA-FIAPARCH</td>
<td>MSE</td>
<td>0.019</td>
<td>0.010</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>MAPE</td>
<td>0.133</td>
<td>0.098</td>
<td>0.083</td>
</tr>
<tr>
<td></td>
<td>LL</td>
<td>17.342</td>
<td>10.255</td>
<td>9.048</td>
</tr>
</tbody>
</table>

We see that the ARFIMA-FIAPARCH specification outperforms the ARFIMA-HYGARCH as well as the ARFIMA-FIGARCH models. Actually, the ARFIMA-FIAPARCH prediction errors are the smallest for all evaluation criteria and all forecast horizons. On the contrary, the ARFIMA-FIGARCH is the worst performing model in terms of out-of-sample predictive accuracy since it produces the largest forecast errors among the set of dual long memory adaptations.
5. Conclusion

This paper attempted to investigate the long memory dynamics in the return series as well as in the volatilities of the daily oil price. For that, we have considered the applicability of three dual long memory adaptations: the ARFIMA-FIGARCH, the ARFIMA-HYGARCH and the ARFIMA-FIAPARCH models. The empirical results obtained show empirical for long memory in the first and second conditional moments.

As a whole, the in-sample diagnostics along with the out-of-sample predictive performances are in favour of an ARFIMA-FIAPARCH modelling process.

It would be worth noting that volatility dynamics are the focus of interest in the vast field of risk management and derivatives pricing. Thus, features of long term persistence, asymmetry and power transformation of the conditional variance should be taken into account when calculating measures of risk, deriving pricing formulas, handling short and long term trading positions.

References


