Saving Rate, Total Factor Productivity and Growth Process for Developing Countries

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Abstract

1 Introduction

The Solow [1957] implies that the TFP is the core factor of economic growth. If the economy bases merely on capital accumulation without technological progress, the diminishing returns on capital accumulation will eventually depresses economic growth to zero. Accordingly, Solowian supporters attribute the miracle economic growths in Newly Industrialized Economies (NIEs) in second half of 20th century to adoption of technologies previously developed by more advanced economies. Pack [1992] suggests "the source of growth in a few Asian economies was their ability to extract relevant technological knowledge from industrial economies and utilize it productively within domestic economy".

Empirically, however, Young [1994, 1995], Kim and Lau [1994, 1996] found that the postwar economic growth of the NIEs was mostly due to growth in input factors (physical capital and labor) with no increase in the total factor productivity. Moreover, the hypothesis of no technical progress cannot be rejected for the East Asian NIEs (Kim and Lau [1994]). Consequently, accumulation of physical and human capital seems to explain the major part of the NIEs’ growth process. Krugman’s [1994] concludes that "it (high growth rate)
was due to forced saving and investment, and long hours of works...So if we are forced to save 40% of our income, and get only two weeks off a year of course a country will growth". Accordingly, due to diminishing returns the lack of technological progress will inevitably bound the growth engine of East Asian NIE.

In the following we will prove that the so-called Solow-Krugman controversy is not a real one. First, we prove that high saving rates may play an important role in "miracle growth" in NIEs in the short and mid terms, but in the long term TFP is the crucial factor of growth as claimed by Krugman. Other things equal, a country with higher saving rate will enjoy higher growth rate in the course of development. Similarly, ceteris paribus, a country with higher rate of technological improvement also enjoys higher growth rate in the process of development. However, the influence of technological improvement dominates in long term. We prove that of two economies which are identical in everything, except for the saving rates and the technological progress, the economy whose rate of technological progress is higher will grow faster in long run, regardless of saving rate.

In this paper we go beyond the so-called Solow-Krugman controversy. We endogenize the growth rate of technological progress (or total factor of productivity, TFP) in a model in section (2). The model proves that in process of development, one country under normal conditions would first invest in physical capital, and then invest in technology which help the country improve TFP. The lag time for investing in technology depends crucially on saving rate and initial level of capital stock. Furthermore, If a country is initially very poor and the marginal benefit from investing in technology at initial stage is low, that country could never invest in technology and their growth would cease in long run. In other words, if effectiveness of technology on the whole economy is too low, a poor country may fall in poverty trap. In this case, foreign capital would be helpful in pushing the economy out of the trap.

To learn about the transitional dynamics to economic growth in developing country we conduct dynamic simulations using a range of parameters values which are conventional in public finance and macroeconomics. We find that

In section (1.1) we revisit Solow model to make it as a benchmarking model for next sections. In section (1.2), section (1.3) we show that TFP and Saving are both important factors for economic growth. However, in the long run, TFP is crucial factor and overrules the role of saving rate. This point is proved in section (1.4). The process of shifting investment from physical capital to technology will be presented in section (2). In order to make our argument more robust, we run a simulation for 100 periods in section (3).

Let us first revisit the Solow model
1.1 The Solow Model (Solow, 1956)

We consider a simple intertemporal growth model for a closed economy.

\[ C_t + S_t = Y_t \]
\[ S_t = sY_t, \text{ } s \text{ is the exogenous saving rate} \]
\[ K_{t+1} = K_t(1-\delta) + I_t \]
\[ L_t = L_0(1+n)^t \]
\[ Y_t = a(1+\gamma)^t K_t^\alpha L_t^{1-\alpha}, \text{ } 0 < \alpha < 1 \]
\[ I_t = S_t \]

\( C_t, S_t, Y_t, K_t, I_t, L_t \) denote respectively the consumption, the saving, the output, the capital stock, the investment and the labour at period \( t \). The labour force grows with an exogenous rate \( n \). The Total Factor Productivity (TFP) grows at rate \( \gamma \). It is easy to solve the model given above. Actually, we have

\[ \forall t, K_{t+1} = (1-\delta)K_t + saK_t^\alpha L_t^{1-\alpha}(1+\gamma)^t \] (1)

We can easily check that there exists a Balanced Growth Path (BGP) with rate \( g \)

\[ (1+g) = (1+n)(1+\gamma)^{\frac{1}{1-\alpha}} \]

On the BGP, we have \( K_t^* = K^*(1+g)^t, \forall t \), where \( K^* = \left( \frac{sa}{\gamma+n} \right)^{\frac{1}{1-\alpha}} L_0 \). Given \( K_0 > 0 \), the path generated by equation (1) satisfies

\[ \frac{K_t}{(1+g)^t} \rightarrow K^* \]

In other words, the path \( \{ K_t \} \) converges to the steady state \( K^* \). It is interesting to notice that the rate of growth \( g \) is positively related to the rate of growth \( \gamma \) of the TFP. Notice also that \( \frac{K_t}{K_{t-1}} \) converges to \( 1+g > 1 \). From equation (1) we have:

\[ \forall t, K_{t+1} = (1-\delta)K_t + saK_t^\alpha L_0^{1-\alpha}(1+\gamma)^t(1+n)^{(1-\alpha)} \] (2)

1.2 On the influence of TFP

Now let us consider two economies which are identical in everything, except for technological progress. The technological progress in economy 1 is \( \gamma \) and in economy 2 is \( \gamma' \) and assume that \( \gamma < \gamma' \). It is obvious that \( g < g' \) and \( K^* > K'^* \). Furthermore, from equation (2) we have: \( K_1 = K_1' \) and \( K_t < K'_t \), \( \forall t > 1 \). For simplicity, we assume \( n = 0 \) and \( L_0 = 1 \).
Define growth rates in these two economies as follows:

\[ \nu_t = \frac{K_t}{K_{t-1}} \quad \text{and} \quad \nu'_t = \frac{K'_t}{K'_{t-1}} \]

We will prove that if \( \gamma < \gamma' \) then \( \nu_t < \nu'_t \), \( \forall t > 1 \). It is obvious to see that \( \nu_1 = \nu'_1 \) and \( \nu_2 < \nu'_2 \).

From equation (2) we have:

\[
\begin{align*}
\frac{K_t}{K_{t-1}} - (1 - \delta) &= sa(1 + \gamma)^t K_{t-1}^{\alpha-1} \\
\frac{K'_t}{K'_{t-1}} - (1 - \delta) &= sa(1 + \gamma)^{t-1} K_{t-2}^{\alpha-1} \\
&\quad \cdot (1 + \gamma) \left( \frac{K_{t-1}}{K_{t-2}} \right)^{\alpha-1}
\end{align*}
\]

Or equivalently:

\[
\nu_t - (1 - \delta) = (1 + \gamma)^2 \nu_{t-1}^{\alpha-1} [\nu_{t-1} - (1 - \delta)] \tag{4}
\]

Let \( \varphi(\nu) = [\nu - (1 - \delta)]\nu^{\alpha-1} \) with \( \nu > 0 \). Then \( \varphi \) is increasing with \( \nu \), since \( \varphi(\nu) = \nu^{\alpha} - (1 - \delta)\nu^{\alpha-1} \) (sum of two increasing functions in \( \nu \)). Since \( \nu_2 < \nu'_2 \), by induction, we get \( \nu_t < \nu'_t \).

### 1.3 On the influence of the saving rate

Again, consider two economies which are identical in everything, except for the saving rates. The saving rate in economy 1 is \( s \) and in economy 2 is \( s' \) and assume that \( s < s' \). It is obvious that \( K^s < K^{s'} \).

We will prove that \( \nu_t < \nu'_t \), \( \forall t > 1 \). It is obvious to see that \( \nu_1 = \nu'_1 \) and \( \nu_2 < \nu'_2 \). We obtain as before:

\[
\nu_t - (1 - \delta) = (1 + \gamma)^2 \nu_{t-1}^{\alpha-1} [\nu_{t-1} - (1 - \delta)] = (1 + \gamma)^2 \varphi(\nu_{t-1})
\]

Since \( \varphi \) is increasing and \( \nu_2 < \nu'_2 \), we get, by induction, \( \nu_t < \nu'_t , \forall t > 1 \).

**Remark 1** From the previous results, one cannot decide, between Solow and Krugman, who was right, who was wrong since both saving rate and technological progress push up the rates of growth. The next section sheds a light on this controversy.
1.4 On the controversy

Again, consider two economies which are identical in everything, except for the saving rates and the technological progress. The saving rate in economy 1 is \( s \) and in economy 2 is \( s' \). The technological progress in economy 1 is \( \gamma \) and in economy 2 is \( \gamma' \). We assume that \( s > s' \) while \( \gamma < \gamma' \). We obtain the following equations for the growth rates.

For economy 1

\[
\nu_t = 1 - \delta + (1 + \gamma)^2 \varphi(\nu_{t-1})
\]

For economy 2

\[
\nu'_t = 1 - \delta + (1 + \gamma')^2 \varphi(\nu'_{t-1})
\]

We obtain \( \nu'_t > \nu_t \) since \( s > s' \). We claim that there exists \( T \) such that for \( t \leq T - 1 \), \( \nu_t > \nu'_t \) and for \( t \geq T \) then \( \nu_t < \nu'_t \). From section 1.1, we know that \( \nu_t \to 1 + g \) and \( \nu'_t \to 1 + g' \) when \( t \to +\infty \). If for any \( t \geq 1 \) we have \( \nu_t \geq \nu'_t \), or equivalently \( \frac{\varphi(\nu_{t-1})}{\varphi(\nu'_{t-1})} \geq \frac{(1+\gamma)^2}{(1+\gamma')^2} \), then letting \( t \) go to infinity we obtain

\[
1 > \frac{(1+g)^2}{(1+g')^2} \geq \frac{(1+\gamma)^2}{(1+\gamma')^2} > 1
\]

which is a contradiction.

Hence there exists \( T \) such that \( \nu_T < \nu'_T \). Since

\[
\nu_{t+1} = (1 - \delta) + (1 + \gamma)^2 \varphi(\nu_t)
\]

\[
\nu'_{t+1} = (1 - \delta) + (1 + \gamma')^2 \varphi(\nu'_t)
\]

we conclude that \( \nu_t < \nu'_t \) for all \( t > T + 1 \), since \( \gamma < \gamma' \).

2 Beyond the Solow-Krugman Controversy

In this section we present a simple model which reconciles the roles of saving rates and technical progress. First, we assume that, at any period \( t \), the saving \( S_t \) will be used to invest in physical capital and to buy some technology \( T_t \) in order to improve the Total Factor Productivity \( \Gamma \) of the next period \( t + 1 \). Let \( \sigma_t \) denote the stock of technology at period \( t \). We then have

\[
S_t = S_{1,t} + T_t
\]

where \( S_{1,t} \) is devoted to physical capital purchase. The technology expenditures will be financed by some tax \( \mu_t Y_t \), that means \( T_t = \mu_t Y_t \). The output, at date
$t + 1$ will be given by $Y_{t+1} = a \Gamma(\sigma_t(1 - \zeta) + T_t)K_t^\alpha_{t+1}$ where $\zeta \in [0, 1]$ is the depreciation rate of the technology stock. The technology stock at $t + 1$ will be $\sigma_{t+1} = \sigma_t(1 - \delta) + \mu_t Y_t$. The dynamics of the capital stock now is

$$K_{t+1} = K_t(1 - \delta) + S_{1,t}$$

$$= K_t(1 - \delta) + (s - \mu_t)Y_t$$

where $Y_t = a \Gamma(\sigma_{t-1}(1 - \zeta) + \mu_{t-1} Y_{t-1})K_t^\alpha$ (5)

We assume that the function $\Gamma$ is differentiable, strictly concave, and it satisfies $\Gamma(0) = 1$, $\Gamma'(0) \leq +\infty$, $\Gamma'(+\infty) = 0$.

We want to maximize the growth rate at each period, i.e. $\frac{Y_t - Y_{t-1}}{Y_{t-1}}$. Since $Y_0$ is given by $aK_0^\alpha$, at period 1, we will maximize $Y_1$ by choosing $\mu_0$, and successively, at period $t$ we will maximize $Y_{t+1}$ by choosing $\mu_t$. It turns out to solve at period $t + 1$ the problem

$$\max \{a \Gamma(\sigma_t(1 - \zeta) + \mu Y_t)(1 - \delta)K_t + (s - \mu)Y_t\}^{\alpha} : \mu \in [0, s]\}$$

Assuming the solution is interior, the FOC is

$$\Gamma'(\sigma_t(1 - \zeta) + \mu Y_t) = \frac{a \Gamma(\sigma_t(1 - \zeta) + \mu Y_t)}{K_t(1 - \delta) + (s - \mu)Y_t}$$

(7)

We can write equation (7) as follows:

$$K_t(1 - \delta) + sY_t = \mu Y_t + \frac{a \Gamma(\sigma_t(1 - \zeta) + \mu Y_t)}{\Gamma'(\sigma_t(1 - \zeta) + \mu Y_t)}$$

The RHS of (7) is increasing in $\mu$ while the LHS is constant, independent of $\mu$. Hence, the solution will $\mu^*$ will be unique. At period $t$, we obtain the optimal value $\mu_t^*$ for $\mu_t$. We then compute $K_{t+1}, Y_{t+1}$. Use (7) to compute $\mu_{t+1}^*$ after replacing $K_t, Y_t$ by $K_{t+1}, Y_{t+1}$, and so on. However the optimal value $\mu_t^*$ may be 0 or $s$. More precisely:

- If $\Gamma'(\sigma_t(1 - \zeta) \leq \frac{a \Gamma(\sigma_t(1 - \zeta))}{K_t(1 - \delta) + sY_t}$ then $\mu_t^* = 0$.

- If $\Gamma'(\sigma_t(1 - \zeta) > \frac{a \Gamma(\sigma_t(1 - \zeta))}{K_t(1 - \delta) + sY_t}$ and $\Gamma'(\sigma_t(1 - \zeta) + sY_t) < \frac{a \Gamma(\sigma_t(1 - \zeta) + sY_t)}{K_t(1 - \delta)}$ then $0 < \mu^* < s$.

- If $\Gamma'(\sigma_t(1 - \zeta) + sY_t) \geq \frac{a \Gamma(\sigma_t(1 - \zeta) + sY_t)}{K_t(1 - \delta)}$ then $\mu^* = s$.

(a) Assume $\Gamma'(0) > \frac{a}{K^\alpha}$. If $K_0$ is very small, then $\mu_0^* = 0$. We then compute $K_1, Y_1$. Use (7) to compute $\mu_1^*$ after replacing $K_0, Y_0$ by $K_1, Y_1$, and so on. However the optimal value $\mu_1^*$ may be 0 or $s$. More precisely:

- If $\Gamma'(\sigma_1(1 - \zeta) \leq \frac{a \Gamma(\sigma_1(1 - \zeta))}{K_1(1 - \delta) + sY_1}$ then $\mu_1^* = 0$.

- If $\Gamma'(\sigma_1(1 - \zeta) > \frac{a \Gamma(\sigma_1(1 - \zeta))}{K_1(1 - \delta) + sY_1}$ and $\Gamma'(\sigma_1(1 - \zeta) + sY_1) < \frac{a \Gamma(\sigma_1(1 - \zeta) + sY_1)}{K_1(1 - \delta)}$ then $0 < \mu^* < s$.

- If $\Gamma'(\sigma_1(1 - \zeta) + sY_1) \geq \frac{a \Gamma(\sigma_1(1 - \zeta) + sY_1)}{K_1(1 - \delta)}$ then $\mu^* = s$. If again,
$K_1$ still is small, we will not invest for period 2 and $K_2 = K_1(1 - \delta) + saK_1^T$, and so on. Actually, there will be a period $T$ such that $K_{T+1} = (K_T(1 - \delta) + sY_T)\Gamma'(0) > \alpha$. Indeed, if we never invest in technology, then the dynamics of $(K_t)$ is

$$K_{t+1} = K_t(1 - \delta) + saK_t^T, \text{ for all } t \geq 0$$

From section (1.1), the sequence will converge to $K^* = (\frac{s}{\alpha})^{\frac{1}{\gamma - \delta}}$. Since we assume $(\frac{s}{\alpha})^{\frac{1}{\gamma - \delta}} = K^*\Gamma'(0) > \alpha$, for $t$ large enough we have $K_{t+1} \Gamma'(0) > \alpha$ and we will invest in technology at this period $t$. We also see that the larger is $s$ the closer is the period where we invest in technology.

(b) If $\Gamma'(0) < K^*$ and if $K_0$ is very small (a very poor country) then this country will never invest in technology for the productivity. This is the case where the technology $\Gamma$ is very bad, i.e. $\Gamma'(0)$ is very small.

(c) Now assume $\Gamma'(0) > \frac{\alpha}{K_0(1 - \delta)}$. If the saving rate $s$ is very small, then we have

$$\Gamma'(0) > \frac{\alpha}{K_0(1 - \delta) + sY_0}$$

$$\Gamma'(sY_0) > \frac{\alpha\Gamma(sY_0)}{K_0(1 - \delta)}$$

In this case, at period 0, the country will totally invest in productivity. This is the case where the quality of the productivity technology $\Gamma$ is very good ($\Gamma'(0)$ is high) and the country has a small rate of saving.

Summing up, in this section, we give an explanation to the empirical results obtain in Kim and Lau (1994), and Lau and Park (2003). However, we go beyond the cases empirically observed in their papers by exhibiting the possibility to invest in productivity even the country is not rich.

Remark 2 The main difference with Bruno, Le Van and Masquin, and Le Van, Nguyen, Nguyen and Luong, is that in these papers the saving rate $s$ is endogenous while here, it is exogenous.

3 On the dynamics of the capital and output trajectories

Based on the model represented in section (2), we run simulations on the dynamics of capital and output in development process. The simulations show how the economy shift into investing in technology and the interaction of saving and technological progress in the course of development.
Calibration

First we restrict the production in Cobb-Douglas form in which labor is assumed unchanged overtime:

\[
Y_{t+1} = a\Gamma(\sigma_t(1 - \zeta) + \mu Y_t)((1 - \delta)K_t + (s - \mu)Y_t)^\alpha \\
K_t = (1 - \delta)K_{t-1} + (s - \mu_{t-1})Y_{t-1} \\
K_0 = 1.5 \text{ (given)}
\]

We normalize the level parameter \(a\) to unity and choose capital-share parameter \(\alpha\) to be 0.3 which accords with majority of empirical studies such as Maddison (1987 table 8).

Second, we choose functional form of technology production as: \(\Gamma(x) = \ln(x + e)\). It is straightforward to see that this function satisfies conditions mentioned in section (2). Parameter \(\lambda\) denotes the efficiency of investing in technology, the higher \(\lambda\) the lower efficiency of investing in technology. To learn the role of efficiency of technology we run two alternative values of \(\lambda\): \(\lambda = 2\) and then \(\lambda = 4\), for each value of saving rate.

Third, the saving rates observed in developing economies, especially in Asian economies such as South Korea, Taiwan, Singapore, Vietnam, China, in their process of development are in range of 0.2 to 0.4. We, hence, choose three alternative values of saving rate: first \(s = 0.2\), then \(s = 0.3\), and \(s = 0.4\).

Fourth, we choose the depreciation rate \(\delta\) and \(\zeta\) to be 5% which is in the range for developing economies\(^1\).

Results:

In this section we would like to examine the trajectories of 6 indicators: rate of investment in technological capital, growth rate, output, capital, share investment in technological capital to total capital, and the ratio of output per capital by 6 different scenarios. The results are presented in appendix in 6 panels. In each panel we present 6 different scenarios for one indicator. Panel A presents the trajectories of rate of investment in technological capital; Panel B presents trajectories of growth rate of output; Panel C present trajectories of Output; Panel D presents trajectories of Capital; Panel E presents trajectories of share investment in technological capital to total capital; and Panel F presents trajectories of the ratio of output per capital.

In each panel, the baseline scenario is presented in top left corner (saving rate \(s = 0.2\) and \(\lambda = 4\)).

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\(^1\)Easterly and Rebelo [1993] use the depreciation rate of 7% uniformly for all countries and all periods, as compared to 0.04 in Nehru and Dhareshwar [1993]
Panel A and Panel E show that, saving and effectiveness technology both play role in decision of investment in technological capital. If technology is good, one economy almost invest in technological capital even in case of low saving. However, if technology is not good enough, the economy would delay their investment in technological capital. In this case, the higher saving rate, the sooner they invest. Furthermore, saving rate and effectiveness of technology are also positively affect the rate of investment in technological capital in transitional period. A country whose higher saving rate and/or better technology tend to invest heavier in transitional period. However, in the long-run the share investment in technological capital to total capital, and rate of investment in this capital tend to converge to a steady state.

Panel B shows that country with higher saving rate will enjoy longer period of high growth rate. Among two economies with the same saving rate, country with better technology grow faster in any period. This implies that catch-up process requires not only better technology but also higher saving rate. Panel C confirms this implication by showing an interesting point for developing countries: saving rate plays an important role in catch-up process. A country with good technology but low saving rate has to take much longer time to catch-up with developed countries. The role of saving will fade out in long run but not in 100 period as shown in this case.

Effectiveness of technology and saving rate also play crucial role in capital accumulation, which can be seen in Panel D. The better technology not only helps country to grow but also to accumulate capital faster.

4 Conclusion

Our models prove that saving and technological progress play essential roles in the course of development. However, the influence of technological improvement dominates in long term. We prove that of two economies which are identical in everything except for the saving rates and the technological progress, the economy whose rate of technological progress is higher will grow faster in long run, regardless of saving rate.

Krugman’s view is correct in the sense that the high saving rate plays an important role in "miracle growth" in NIEs. Our simulation results show that in transitional stage saving always play an important role in growth process. A country with higher saving rate will enjoy longer period of high growth rate. This effect of high saving rate will die out in the long-run. However higher
saving rate is crucial for developing country to catch-up with developed ones in limited period.

Furthermore, the decision to invest in technological capital depends on the initial stock of capital and effectiveness of technology. At initial stages when a country is still poor, they need not investing in technological capital but physical capital. After some stages, one country will invest in technological capital if technology is good enough. The proportion of this investment in technological capital increases sharply in short time and then slowly converges to a steady state. Country with higher saving rate tend to invest in technological capital sooner, this country also maintain higher growth rate of output in transitional period. Finally, one country may fall into poverty trap if they are too poor to start up and the technology is not good enough. At this point, international aids and foreign investment are essential for them to escape the trap.

References


[31] National Science Council (2007), *Indicators of Science and Technology Taiwan*, Taiwan.


5 Appendix
Panel A: Rate of investment in technology capital

\[ \lambda = 4 \quad \lambda = 2 \]

\( s = 0.2 \)

\( s = 0.3 \)

\( s = 0.4 \)
Panel B: Rate of growth

\[ \lambda = 4 \]

\[ \lambda = 2 \]

\[ s = 0.2 \]

\[ s = 0.3 \]

\[ s = 0.4 \]
Panel C: Output

\[ \lambda = 4 \]

\[ \lambda = 2 \]

\[ s = 0.2 \]

\[ s = 0.3 \]

\[ s = 0.4 \]
Panel D: Capital

\[ \lambda = 4 \]

\[ s = 0.2 \]

\[ \lambda = 2 \]

\[ s = 0.3 \]

\[ s = 0.4 \]
Panel E: Proportion of investment in technology to the total

\[ \lambda = 4 \quad \text{and} \quad \lambda = 2 \]

\[ s = 0.2, \quad s = 0.3, \quad s = 0.4 \]
Panel F: Ratio of Output per capital

\( \lambda = 4 \)

\( \lambda = 2 \)

\( s = 0.2 \)

\( s = 0.3 \)

\( s = 0.4 \)